Flight Dynamics of Elastic Vehicles - With Emphasis on Modeling & Real-Time Simulation

Dr. David K. Schmidt
Professor Emeritus
University of Colorado

dschmidt@uccs.edu

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Outline

- Background & Motivation
- Dynamic Modeling

  Equations of Motion
  Model Structure
- Modal Analysis
- Simulation Issues
- Flight-Control Issues
- Summary and Conclusions

Main References

- Schmidt, Modern Flight Dynamics, McGraw-Hill, 2012. (Short Course)

- Waszak and Schmidt, "Flight Dynamics of Aeroelastic Vehicles," AIAA JAC, Vol. 25, No. 6, 1988.

- Waszak, Buttrill, and Schmidt, Modeling and Model Simplification of Aeroelastic Vehicles: An Overview, NASA TM-107691,1992.

- Waszak, Davidson, and Schmidt, D.K., "A [Real-Time] Simulation Study of the Flight Dynamics of Elastic Aircraft," NASA CR 4102, Vols I and II, Dec., 1987.

- Schmidt and Raney, "Modeling and Simulation of Flexible Flight Vehicles," JGC&D, Vol. 24, No. 3, 2001.

- Schwanz, Cerra, and Blair, "Dynamic Modeling Uncertainty Affecting Control System Design," AIAA Paper No. 84-1057-CP, Dynamics Specialists Conference, 1984.

Motivation

It has long been known that static-elastic deformation can significantly influence

Static Stability - Vehicle Trim Configuration - Control Power - Handling Qualities

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But it is also apparent that dynamic-elastic effects can significantly influence

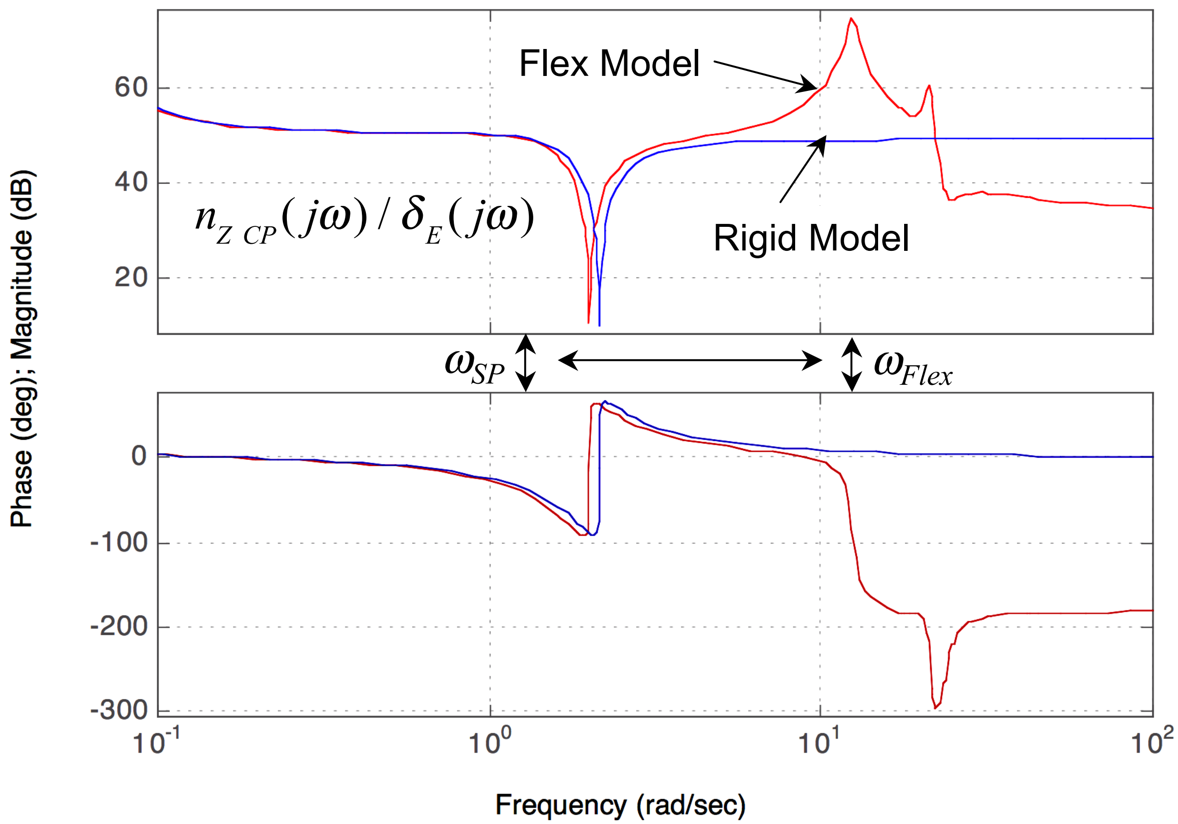
Vehicle “rigid-body” dynamics - coupled rigid-body/elastic DOFs - e.g., B2 Resid. Pitch Osc.
Vehicle dynamic stability - e.g., X-29 “body-freedom flutter”
Ride and handling qualities - e.g. B1, XB-70, HSCT
Achievable bandwidth and stability margins of the flight-control system
- c.f., Schwanz, et al, AIAA 84-1057-CP
Complexity (cost) of the flight-control/structural-mode-control systems - many

XB-70

X-29

Example Frequency Response
Large, High-Speed Aircraft

Cockpit Vertical-Acceleration Response Key in Both Handling and Ride Qualities



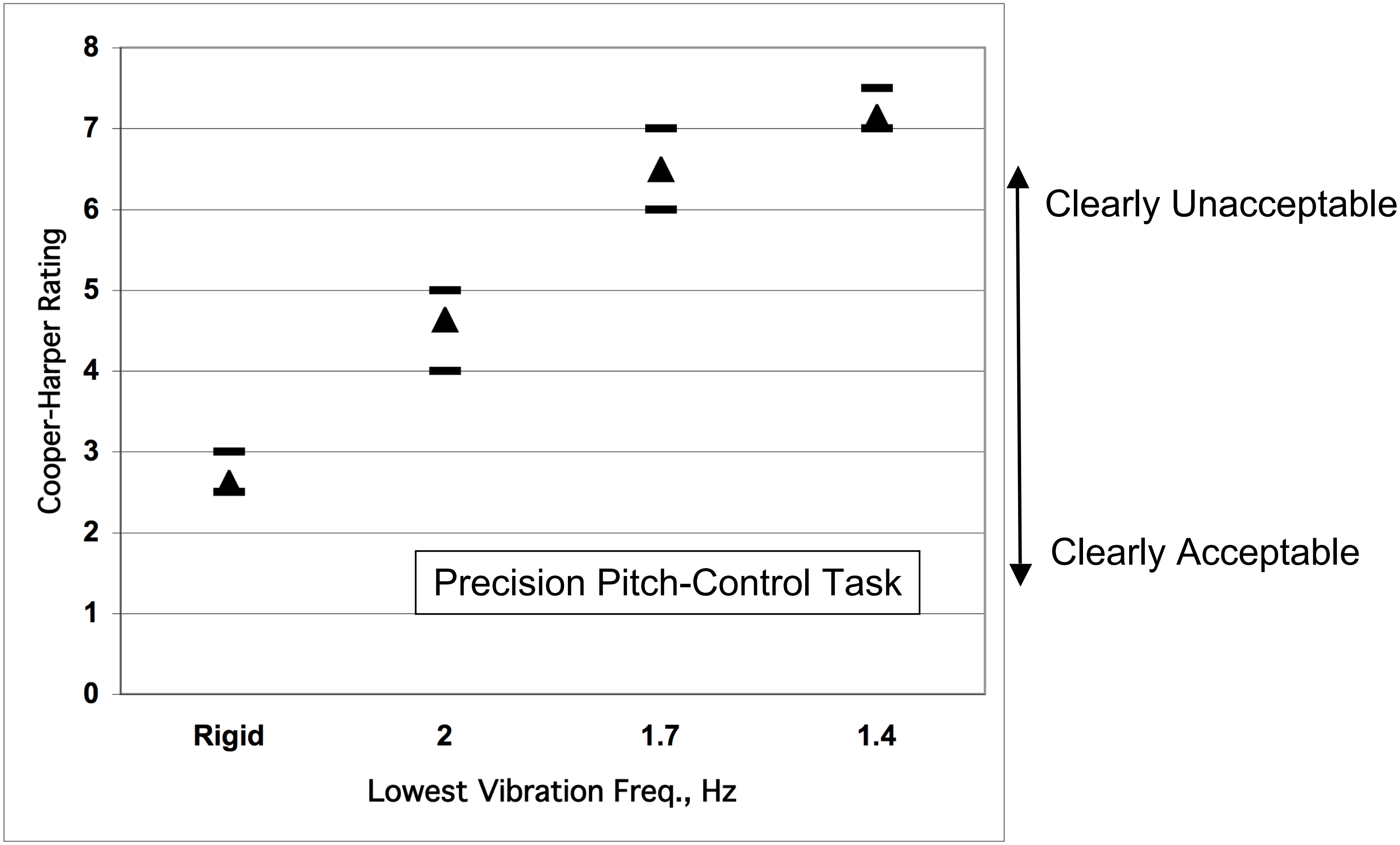
Flex Model -
First 4 sym. modes

- "Rigid-body" dynamics (e.g., ω_{SP}, ζ_{SP}) affected
- Significant amplitude and phase differences above ω_{SP} not captured simply through static-elastic corrections to rigid-body aerodynamics

Waszak & Schmidt, 1988.

Effects on Vehicle Handling Qualities

Real-Time, Motion Simulation Results



Significant degradation not explained by only changes in rigid-body modal parameters

Dynamic-elastic effects significant

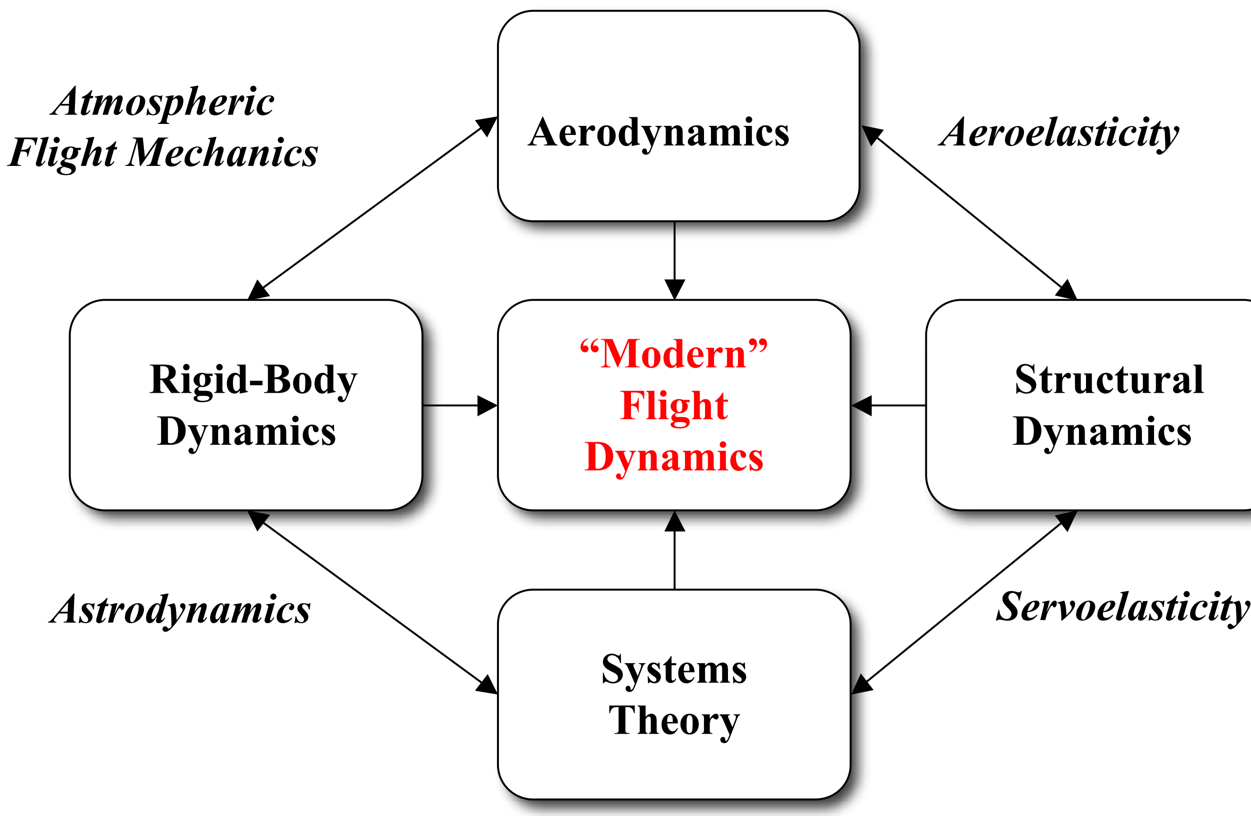
Waszak, Davidson, and Schmidt, 1987.

Prognosis

As the frequencies of the elastic modes are further reduced (e.g., HALE vehicles), and/or increased performance of flight-control systems is required (e.g., reduced aerodynamic stability), elastic effects likely to become even more significant in HQ and flight-control design. (e.g., DARPA Vulture program)

Helios

Disciplinary Morphology



A broad, integrated perspective is needed

Two Disciplinary Cultures

Flight Dynamics Culture

- Charge -Tailor the vehicle dynamics:

  Handling & performance
  Feedback stability augmentation
  Real-time, pilot-in-the loop simulation

- Key dynamics (e.g.):

$$\frac{\theta(s)}{\delta(s)} = \frac{K\left(s + 1/T_{\theta_1}\right)\left(s + 1/T_{\theta_2}\right)}{\left(s^2 + 2\zeta_P\omega_P s + \omega_P^2\right)\left(s^2 + 2\zeta_{SP}\omega_{SP} s + \omega_{SP}^2\right)}$$

- RB modal characteristics are critical

- Truncate elastic degrees of freedom

Aeroelasticity Culture

- Charge - Provide structural integrity:

  Mitigate against flutter, divergence
  Tailoring & active structural-mode control
  Fast-time simulation, tunnel tests

- Key dynamics:

$$\left[\mathbf{M}\right]\ddot{\mathbf{q}} + \left[\mathbf{K}\right]\mathbf{q} = \sum_{i=1}^{m} \mathbf{F}_{aero_i}(\mathbf{q})$$

- Extensive computational analysis

- Truncate rigid-body degrees of freedom

Cross-Disciplinary Challenge

Modeling the Flight Dynamics of Elastic Aircraft

• Several overall approaches could be taken - but keep eye on the prize

➔ Want to apply models to real-time simulation, HQ, and flight-control design rather than flutter analysis, for example

• Desire model structure compatible with classical rigid-body models

➔ Want to be compatible with flight-dynamics/flight-simulation applications

• Time-domain (state-variable) format preferred

• Unsteady aerodynamics may or may not be critical - low reduced frequencies

Modeling Approach

➔ Assume n in-vacuo, unrestrained vibration frequencies ω_i, mode shapes $\mathbf{v}_i$, and generalized masses $\mathcal{M}_i$ are available to describe the flexible structure.

The elastic deformation of the vehicle at (x,y,z) may then be described by

$$\boxed{\mathbf{d}_E(x,y,z,t) = \sum_{i=1}^{n} \mathbf{v}_i(x,y,z)\eta_i(t)} \qquad (4.16)$$

where $\eta_i(t)$ = generalized modal displacement of the i'th vibration mode

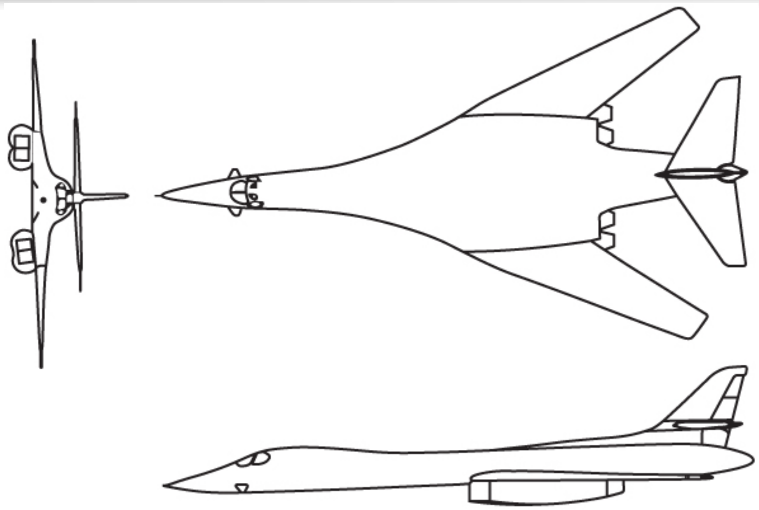
$\mathbf{v}_i(x,y,z)$ = mode shape (vector) of the i'th vibration mode

➔ Require all mode shapes - rigid-body and vibration - to be mutually orthogonal w.r.t. the mass matrix.

➔ Assume elastic displacements sufficiently small such that inertias are constant.

➔ Require the origin of the vehicle-fixed frame to be at the vehicle's instantaneous *cm*

Example Structural Description - Large Flexible Aircraft



Vehicle Geometry

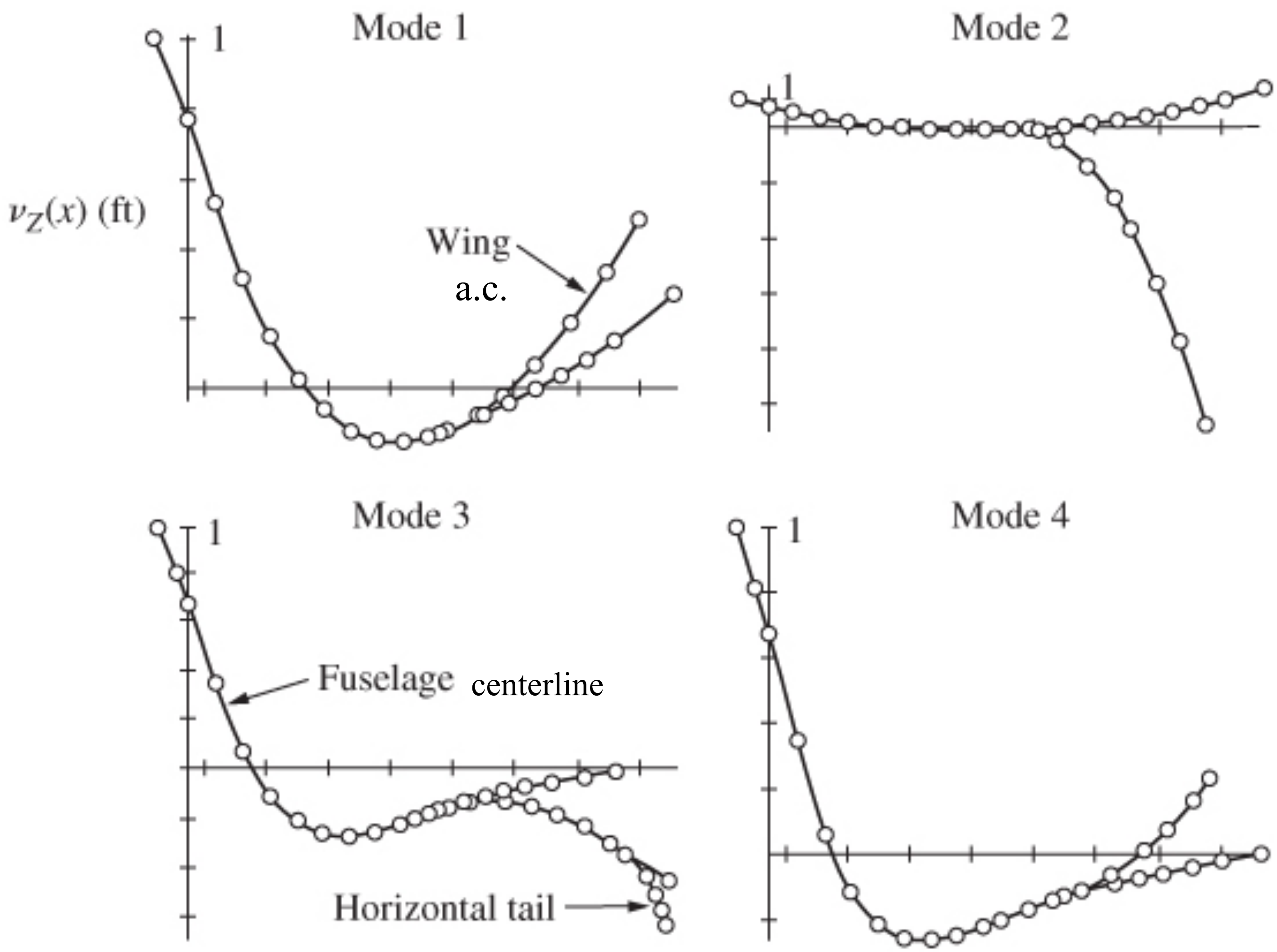
Relevant Data

Wing Geometry	$S_W = 1950\ \text{ft}^2$ $\bar{c}_W = 15.3\ \text{ft}$ $b_W = 70\ \text{ft}$ $\Lambda_{LE} = 65\ \text{deg}$	Inertias	$I_{XX} = 9.5\text{x}10^5\ \text{sl-ft}^2$ $I_{YY} = 6.4\text{x}10^6\ \text{sl-ft}^2$ $I_{ZZ} = 7.1\text{x}10^6\ \text{sl-ft}^2$ $I_{xz} = -52{,}700\ \text{sl-ft}^2$
Weight	$W = 288{,}000\ \text{lb}$	Vehicle Length	143 ft
Modal Generalized Masses	$\mathcal{M}_1 = 184\ \text{sl-ft}^2$ $\mathcal{M}_2 = 9587\ \text{sl-ft}^2$ $\mathcal{M}_3 = 1334\ \text{sl-ft}^2$ $\mathcal{M}_4 = 436{,}000\ \text{sl-ft}^2$	Modal Frequencies	$\omega_1 = 12.6\ \text{rad/sec}$ $\omega_2 = 14.1\ \text{rad/sec}$ $\omega_3 = 21.2\ \text{rad/sec}$ $\omega_4 = 22.1\ \text{rad/sec}$

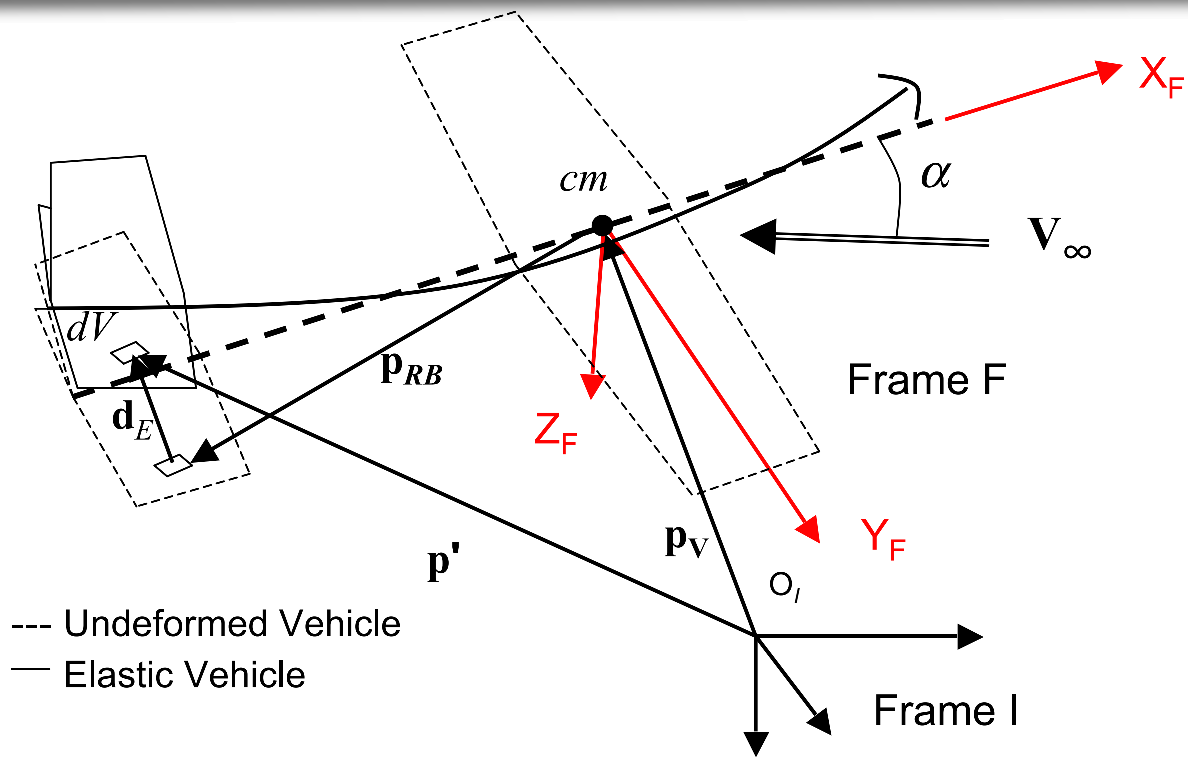
NAA Modal Data Package, 1971.

Example Vibration Mode Shapes

(Wing Twist Not Shown)



Coordinate Frames and Generalized Coordinates



For rigid vehicle

Frame F = fixed fuselage axis

With deformation

Frame F is fixed only in the undeformed vehicle

External (e.g., aero) forces expressed in Frame F

We'll apply Lagrange's equation, using generalized forces

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial T}{\partial \mathbf{q}} + \frac{\partial U}{\partial \mathbf{q}} = \mathbf{Q}^T = \frac{\partial(\delta W)}{\partial(\delta \mathbf{q})} \tag{4.1}$$

And select generalized coordinates $\mathbf{q} = \{X_I \quad Y_I \quad Z_I \quad \phi \quad \theta \quad \psi \quad \eta_i, i = 1, 2, \cdots\}$

Resulting Equations of Motion

(See MFD for Details)

Letting the forces and moments (Frame F) arise from aerodynamic and propulsive effects

Translational Equations
(same as rigid vehicle)

$$m\left(\dot{U} - VR + WQ\right) = -mg\sin\theta + F_{A_X} + F_{P_X}$$
$$m\left(\dot{V} + UR - WP\right) = mg\cos\theta\sin\phi + F_{A_Y} + F_{P_Y} \qquad (4.65)$$
$$m\left(\dot{W} - UQ + VP\right) = mg\cos\theta\cos\phi + F_{A_Z} + F_{P_Z}$$

Rotational Equations

(same as rigid vehicle)

$$I_{xx}\dot{P} - \left(I_{yy} - I_{zz}\right)QR - I_{xy}\left(\dot{Q} - PR\right) - I_{yz}\left(Q^2 - R^2\right) - I_{xz}\left(\dot{R} + PQ\right) = L_A + L_P$$
$$I_{yy}\dot{Q} + \left(I_{xx} - I_{zz}\right)PR - I_{xy}\left(\dot{P} + QR\right) - I_{yz}\left(\dot{R} - PQ\right) + I_{xz}\left(P^2 - R^2\right) = M_A + M_P$$
$$I_{zz}\dot{R} + \left(I_{yy} - I_{xx}\right)PQ + I_{xy}\left(Q^2 - P^2\right) - I_{yz}\left(\dot{Q} + PR\right) - I_{xz}\left(\dot{P} - QR\right) = N_A + N_P$$

(4.82)

Aeroelastic Equations
(new) $\mathbf{P}$ = pressure distribution

$$\ddot{\eta}_i + \omega_i^2\eta_i = \frac{Q_i}{\mathcal{M}_i} = \frac{1}{\mathcal{M}_i}\int_{Area}\mathbf{P}\left(x,y,z\right)\bullet\mathbf{v}_i\left(x,y,z\right)dS \quad i = 1\cdots n \qquad (4.88)$$

Also, note that

$$\mathbf{p}' = \mathbf{p}_V + \mathbf{p}_{RB} + \sum_{i=1}^{n}\mathbf{v}_i(x,y,z)\eta_i(t)$$

We have the inertial position of any point on the vehicle (e.g., a sensor)

Aerodynamic Coefficients - Rigid and Elastic

Now the aero forces and moments are affected by both "rigid-body" and elastic motion, so let

$$F_{A_X} = F_{A_{X_R}} + F_{A_{X_E}}, \quad L_A = L_{A_R} + L_{A_E}$$
$$F_{A_Y} = F_{A_{Y_R}} + F_{A_{Y_E}}, \quad M_A = M_{A_R} + M_{A_E}$$
$$F_{A_Z} = F_{A_{Z_R}} + F_{A_{Z_E}}, \quad N_A = N_{A_R} + N_{A_E}$$

And for example, let the aero pitching moment be expressed as

$$M_A = q_\infty S_W \bar{c}_W \left(C_{M_{Rigid}} + C_{M_{Elastic}} \right)$$

where

$$C_{M_{Rigid}} = C_{M_0} + C_{M_\alpha}\alpha + C_{M_q} q + C_{M_{\dot\alpha}}\dot\alpha + C_{M_\delta}\delta$$

and

$$\boxed{C_{M_{Elastic}} = \sum_{i=1}^{n} \left(C_{M_{\eta_i}} \eta_i + C_{M_{\dot\eta_i}} \dot\eta_i \right)}$$

Likewise, let the generalized force on the i'th elastic DOF be

$$Q_i = q_\infty S_W \bar{c}_W \left(C_{Q_{i\,Rigid}} + C_{Q_{i\,Elastic}} \right)$$

where

$$C_{Q_{i\,Rigid}} = C_{Q_{i\,0}} + C_{Q_{i\,\alpha}}\alpha + C_{L_{\dot\alpha}}\dot\alpha + C_{Q_{i\,\beta}}\beta + C_{Q_{i\,p}} p + C_{Q_{i\,q}} q + C_{Q_{i\,r}} r + \sum_{j=1}^{m} C_{Q_{i\,\delta_j}} \delta_j$$

and

$$C_{Q_{i\,Elastic}} = \sum_{j=1}^{n} \left(C_{Q_{i\,\eta_j}} \eta_j + C_{Q_{i\,\dot\eta_j}} \dot\eta_j \right)$$

Sample Expressions for Elastic Coefficients

Using strip theory we may gain some gain insight regarding coefficients.

Considering a vehicle with conventional geometry we have, for example

$$C_{Q_{i_\alpha}} = \frac{-1}{S_W \bar{c}_W}\left(\int_{-b_W/2}^{b_W/2} c_{l_{\alpha_W}}(y) \nu_{Z_{i_W}}(y) c_W(y)\,dy + \frac{q_H}{q_\infty} \int_{-b_H/2}^{b_H/2} c_{l_{\alpha_H}}(y) \left(1 - \frac{d\varepsilon_H}{d\alpha_W}\right) \nu_{Z_{i_H}}(y) c_H(y)\,dy \right) \tag{7.94}$$

$$C_{Q_{i_{\eta_j}}} = \frac{-1}{S_W \bar{c}_W}\left(\begin{array}{c} \displaystyle\int_{-b_W/2}^{b_W/2} c_{l_{\alpha_W}}(y) \nu'_{Z_{j_W}}(y) \nu_{Z_{i_W}}(y) c_W(y)\,dy - \frac{q_H}{q_\infty} \int_0^{b_V} c_{l_{\alpha_V}}(z) \nu'_{Y_{j_V}}(z) \nu_{Y_{i_V}}(z) c_V(z)\,dz \\ \displaystyle + \frac{q_H}{q_\infty} \int_{-b_H/2}^{b_H/2} c_{l_{\alpha_H}}(y) \left(\nu'_{Z_{j_H}}(y) - \frac{d\varepsilon_H}{d\alpha_W} \nu'_{Z_{j_W}}(y) \right) \nu_{Z_{i_H}}(y) c_H(y)\,dy \end{array} \right) \tag{7.95}$$

Where $\nu_{Z_i}(y)$ = z displacement mode shape of mode i evaluated along wing or tail span y

$\nu'_{Z_i}(y)$ = slope of z displacement mode shape of mode i evaluated along wing or tail span y

Example - Large, High Speed Study Vehicle

Additional force and moment acting on RB DOF's

$$F_{A_{Z_E}} = q_\infty S_W \left(-0.029\eta_1 + 0.306\eta_2 + 0.015\eta_3 - 0.014\eta_4 - \frac{0.658}{V_\infty}\dot{\eta}_1 + \frac{7.896}{V_\infty}\dot{\eta}_2 + \frac{0.461}{V_\infty}\dot{\eta}_3 - \frac{0.132}{V_\infty}\dot{\eta}_4 \right)$$

$$M_{A_E} = q_\infty S_W \bar{c}_W \left(-0.032\eta_1 - 0.025\eta_2 + 0.041\eta_3 - 0.018\eta_4 - \frac{1.184}{V_\infty}\dot{\eta}_1 + \frac{9.409}{V_\infty}\dot{\eta}_2 + \frac{1.316}{V_\infty}\dot{\eta}_3 - \frac{0.395}{V_\infty}\dot{\eta}_4 \right)$$

Generalized forces acting on elastic DOF's

$$Q_1 = q_\infty S_W \bar{c}_W \begin{pmatrix} -0.0149\alpha - \frac{0.726}{V_\infty} Q - 0.0128\delta_E + 5.85\times 10^{-5}\eta_1 + 4.21\times 10^{-3}\eta_2 + 2.91\times 10^{-4}\eta_3 \\ +2.21\times 10^{-5}\eta_4 - \frac{0.0032}{V_\infty}\dot{\eta}_1 + \frac{0.0665}{V_\infty}\dot{\eta}_2 - \frac{0.0048}{V_\infty}\dot{\eta}_3 - \frac{0.0004}{V_\infty}\dot{\eta}_4 \end{pmatrix}$$

$$Q_2 = q_\infty S_W \bar{c}_W \begin{pmatrix} 0.0258\alpha + \frac{0.089}{V_\infty} Q - 0.0642\delta_E - 9.0\times 10^{-5}\eta_1 - 9.22\times 10^{-2}\eta_2 + 1.44\times 10^{-3}\eta_3 \\ -1.32\times 10^{-4}\eta_4 - \frac{0.0015}{V_\infty}\dot{\eta}_1 - \frac{2.277}{V_\infty}\dot{\eta}_2 + \frac{0.1494}{V_\infty}\dot{\eta}_3 + \frac{0.0031}{V_\infty}\dot{\eta}_4 \end{pmatrix}$$

$$Q_3 = q_\infty S_W \bar{c}_W \begin{pmatrix} 0.0149\alpha + \frac{0.304}{V_\infty} Q + 0.0256\delta_E + 3.55\times 10^{-4}\eta_1 + 1.97\times 10^{-3}\eta_2 - 3.46\times 10^{-4}\eta_3 \\ +9.68\times 10^{-6}\eta_4 + \frac{0.0050}{V_\infty}\dot{\eta}_1 + \frac{0.0320}{V_\infty}\dot{\eta}_2 - \frac{0.0001}{V_\infty}\dot{\eta}_3 - \frac{0.0004}{V_\infty}\dot{\eta}_4 \end{pmatrix}$$

$$Q_4 = q_\infty S_W \bar{c}_W \begin{pmatrix} 3.35\times 10^{-5}\alpha + 0.0Q + 1.5\times 10^{-4}\delta_E + 1.20\times 10^{-4}\eta_1 + 3.37\times 10^{-3}\eta_2 + 1.44\times 10^{-4}\eta_3 \\ +1.77\times 10^{-3}\eta_4 - \frac{0.0011}{V_\infty}\dot{\eta}_1 + \frac{0.0317}{V_\infty}\dot{\eta}_2 - \frac{0.0100}{V_\infty}\dot{\eta}_3 + \frac{0.6112}{V_\infty}\dot{\eta}_4 \end{pmatrix}$$

MFD Sec. 7.9

Dynamic Model of the Elastic Aircraft

Elastic Effects

Rigid-Body Translation of cm

$$m\left(\dot{U} - VR + WQ\right) = -mg\sin\theta + \left(F_{A_{X_R}} + F_{A_{X_E}}\right) + F_{P_X}$$

$$m\left(\dot{V} + UR - WP\right) = mg\cos\theta\sin\phi + \left(F_{A_{Y_R}} + F_{A_{Y_E}}\right) + F_{P_Y} \quad (7.98)$$

$$m\left(\dot{W} - UQ + VP\right) = mg\cos\theta\cos\phi + \left(F_{A_{Z_R}} + F_{A_{Z_E}}\right) + F_{P_Z}$$

Rigid-Body Rotation of Frame F

$$I_{xx}\dot{P} - \left(I_{yy} - I_{zz}\right)QR - I_{xz}\left(\dot{R} + PQ\right) = \left(L_{A_R} + L_{A_E}\right) + L_P$$

$$I_{yy}\dot{Q} + \left(I_{xx} - I_{zz}\right)PR + I_{xz}\left(P^2 - R^2\right) = \left(M_{A_R} + M_{A_E}\right) + M_P$$

$$I_{zz}\dot{R} + \left(I_{yy} - I_{xx}\right)PQ - I_{xz}\left(\dot{P} - QR\right) = \left(N_{A_R} + N_{A_E}\right) + N_P \quad (7.100)$$

Elastic Deformation

$$\ddot{\eta}_i + 2\zeta_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = \frac{1}{\mathcal{M}_i}\left(Q_{i_R} + Q_{i_E}\right), \quad i = 1\cdots n \quad (7.102)$$

- Identical form to that of the rigid vehicle, with added elastic components
- Applicable to real-time simulation

On Static-Elastic Corrections - Residualization

Ref. Sec. 7.11

Assuming locally-linear aero, the previous non-linear equations of motion may be written as

$$\mathbf{M}\dot{\mathbf{x}}_R = \mathbf{f}_R\left(\mathbf{x}_R, T\right) + \boxed{\mathbf{A}_R\mathbf{x}_R + \begin{bmatrix}\mathbf{A}_{R\eta} & \mathbf{A}_{R\dot{\eta}}\end{bmatrix}\mathbf{x}_E + \mathbf{B}_R\mathbf{u}}$$

$$\dot{\mathbf{x}}_E = \begin{bmatrix}\mathbf{0} \\ \mathbf{A}_{ER}\end{bmatrix}\mathbf{x}_R + \begin{bmatrix}\mathbf{0} & \mathbf{I} \\ \mathbf{A}_\eta & \mathbf{A}_{\dot{\eta}}\end{bmatrix}\mathbf{x}_E + \begin{bmatrix}\mathbf{0} \\ \mathbf{B}_E\end{bmatrix}\mathbf{u} \qquad (7.126)$$

Aero model of forces and moments

where,

$$\mathbf{x}_R^T = \begin{bmatrix}U & \alpha & Q & \beta & P & R\end{bmatrix}, \quad \mathbf{x}_E^T = \begin{bmatrix}\eta_1 & \cdots & \eta_n & \dot{\eta}_1 & \cdots & \dot{\eta}_n\end{bmatrix}, \quad \mathbf{u}^T = \begin{bmatrix}\delta_E & \delta_A & \delta_R\end{bmatrix}$$

Residualizing the elastic states ($\dot{\mathbf{x}}_E = \mathbf{0}$), yields the static-elastic constraint and reduced-order model. (7.127) (7.128)

$$\eta_0 = -\mathbf{A}_\eta^{-1}\left(\mathbf{A}_{ER}\mathbf{x}_R + \mathbf{B}_E\mathbf{u}\right) \quad \Rightarrow \quad \mathbf{M}\dot{\mathbf{x}}_R = \mathbf{f}_R\left(\mathbf{x}_R, T\right) + \underbrace{\left(\mathbf{A}_R - \mathbf{A}_{R\eta}\mathbf{A}_\eta^{-1}\mathbf{A}_{ER}\right)\mathbf{x}_R + \left(\mathbf{B}_R - \mathbf{A}_{R\eta}\mathbf{A}_\eta^{-1}\mathbf{B}_E\right)}_{\text{Static-elastic aero model}}\mathbf{u}$$

Only the rigid-body degrees of freedom are included in the reduced-order dynamic model here.

Again using the example of the large flexible aircraft,

$$C_{M_\alpha} = -1.5 + \Delta C_{M_\alpha}, \quad \Delta C_{M_\alpha} = 0.23 \text{ /rad}, \quad C_{M_q} = -0.4 + \Delta C_{M_q}, \quad \Delta C_{M_q} = 0.02 \text{ sec} \qquad \text{(Example 7.3)}$$

$$C_{M_{\delta_E}} = -2.58 + \Delta C_{M_{\delta_E}}, \quad \Delta C_{M_{\delta_E}} = 0.22 \text{ /rad}$$

These are static-elastic corrections to the rigid-body stability derivatives - destabilizing

Structure of the Linearized Model Longitudinal Dynamics

Defining appropriate elastic dimensional stability derivatives, we have a dynamic model of the following form (assumes $X_{\dot\alpha} = Z_{\dot\alpha} = M_{\dot\alpha} = \gamma_0 = 0$ for simplicity)

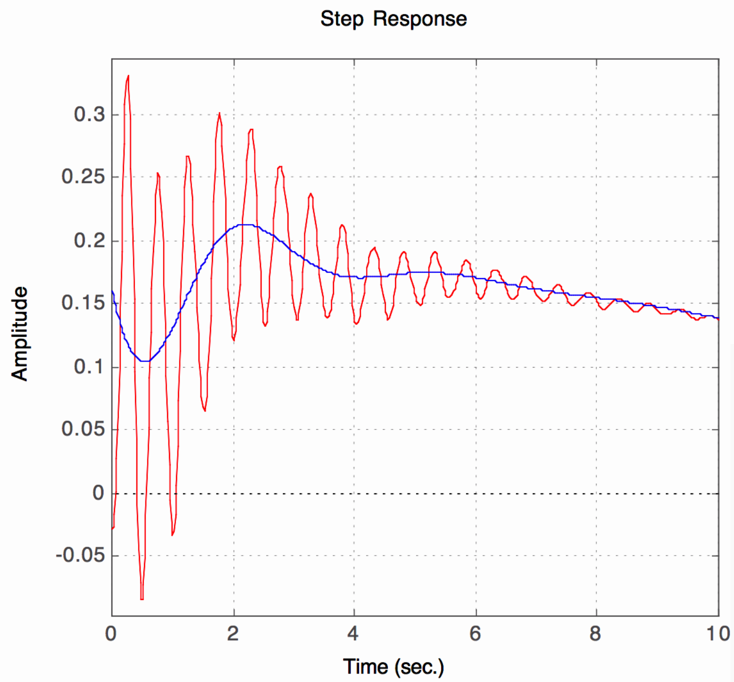
$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{x}^T = \left[u \quad \alpha \quad \theta \quad q \; \vdots \; \eta_1 \quad \dot\eta_1 \quad \cdots \quad \eta_n \quad \dot\eta_n \right], \qquad \mathbf{u}^T = \left[\delta_E \quad \delta T \right]$$

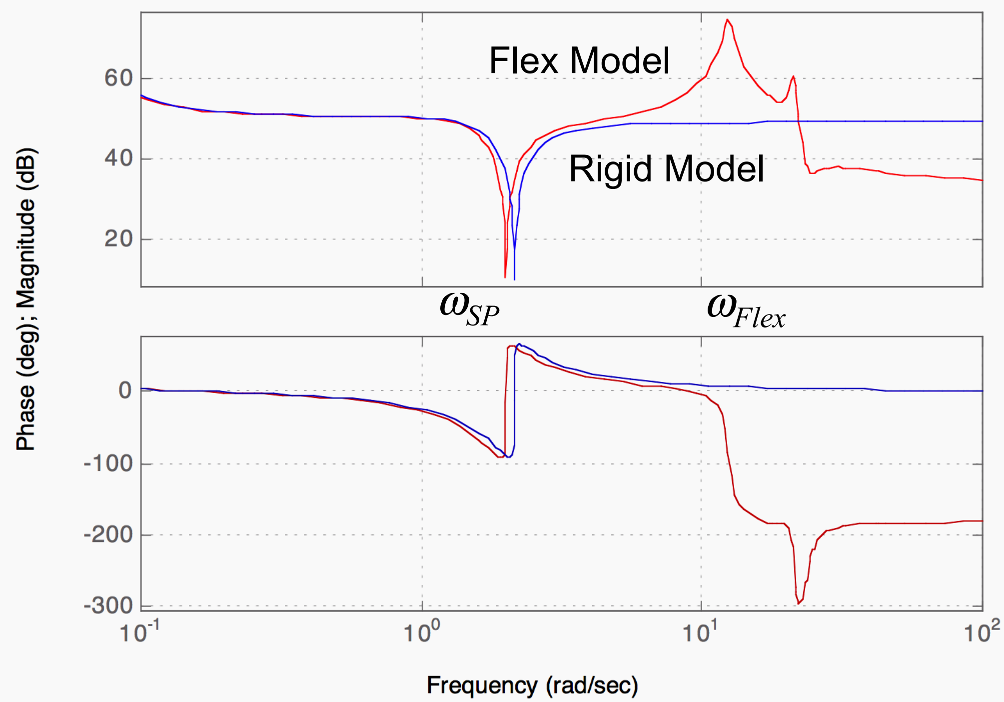
$$\mathbf{A} = \left[\begin{array}{cccc|ccccc}
\bar X_u & X_\alpha & -g & X_q & X_{\eta_1} & X_{\dot\eta_1} & \cdots & X_{\eta_n} & X_{\dot\eta_n} \\
\frac{\bar Z_u}{U_0} & \frac{\bar Z_\alpha}{U_0} & 0 & 1 & \frac{Z_{\eta_1}}{U_0} & \frac{Z_{\dot\eta_1}}{U_0} & \cdots & \frac{Z_{\eta_n}}{U_0} & \frac{Z_{\dot\eta_n}}{U_0} \\
0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\
\bar M_u & \bar M_\alpha & 0 & M_q & M_{\eta_1} & M_{\dot\eta_1} & \cdots & M_{\eta_n} & M_{\dot\eta_n} \\
\hline
0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\
\Xi_{1_u} & \Xi_{1_\alpha} & 0 & \Xi_{1_q} & \Xi_{1_\eta} - \omega_1^2 & \Xi_{1_{\dot\eta}} - 2\zeta_1\omega_1 & \cdots & \Xi_{1_\eta} & \Xi_{1_{\dot\eta}} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\
\Xi_{n_u} & \Xi_{n_\alpha} & 0 & \Xi_{n_q} & \Xi_{n_\eta} & \Xi_{n_{\dot\eta}} & \cdots & \Xi_{n_\eta} - \omega_n^2 & \Xi_{n_{\dot\eta}} - 2\zeta_n\omega_n
\end{array} \right] \qquad
\mathbf{B} = \left[\begin{array}{cc}
X_{\delta_E} & X_T \\
\frac{Z_{\delta_E}}{U_0} & \frac{Z_T}{U_0} \\
0 & 0 \\
M_{\delta_E} & M_T \\
\hline
0 & 0 \\
\Xi_{1_{\delta_E}} & \Xi_{1_T} \\
\vdots & \vdots \\
0 & 0 \\
\Xi_{n_{\delta_E}} & \Xi_{n_T}
\end{array} \right]$$

Model structure exposes subsystems, aero coupling effects and vib. freq. & damping
Applicable to dynamic analysis and control-system design

Comparison of Rigid vs Flexible Models
Vertical Acceleration Responses - Example Vehicle



Step Responses
$n_{Z\,CP}$ (g's)
$\delta_E = 1$ deg

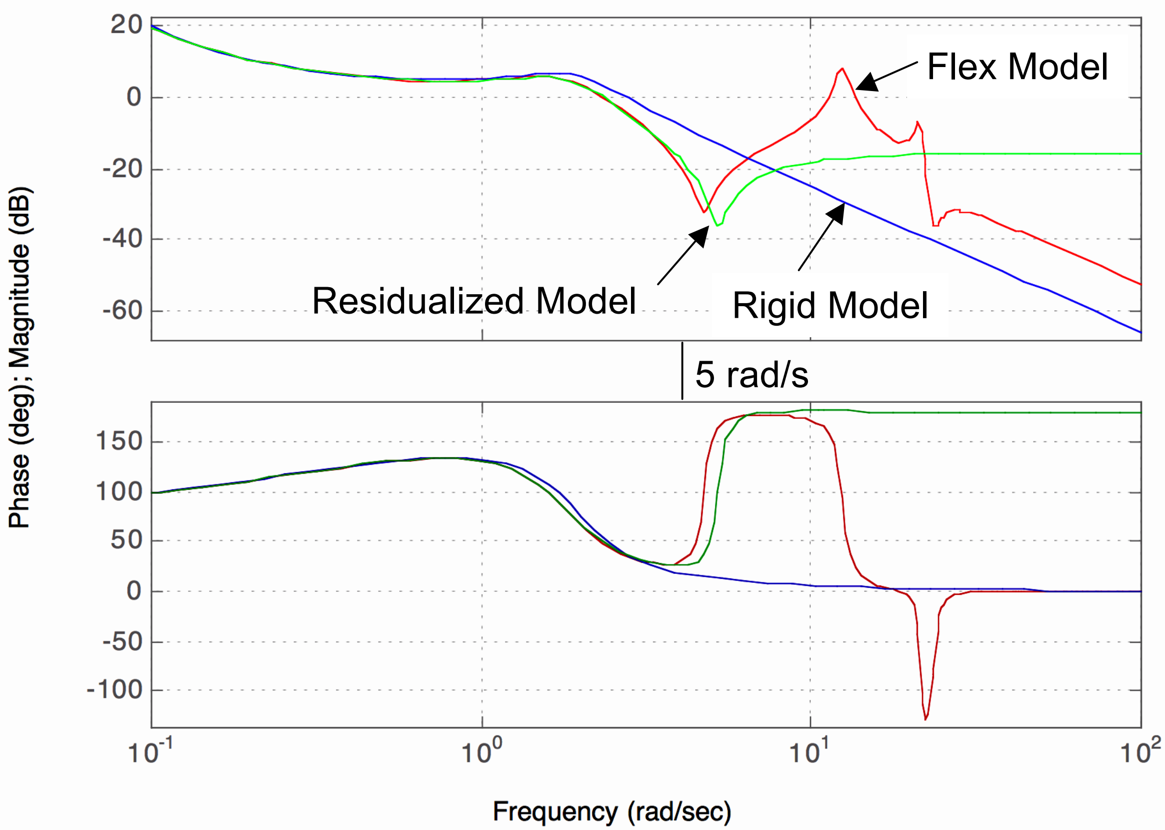


Frequency Responses
$n_{Z\,CP}$ (ft/s^2)
δ_E (rad)

Note: Rigid & Resid, very similar

Model Comparisons - Continued
Pitch-Attitude Response - Example Vehicle

$$\theta_{CP}(j\omega)/\delta_E(j\omega)$$



Key HQ Parameters

Model	ω_{SP} (rad/s)	ζ_{SP}
Rigid	1.972	0.348
Resid.	1.838	0.355
Flex	1.838	0.346

Model	ω_P (rad/s)	T_P(s)
Rigid	0.066	-0.0002
Resid.	0.066	-0.0002
Flex	0.066	-0.0002

- Residualized Model Improved Over Rigid
- But Clearly Inadequate Above ~ 5 rad/s
- Well Within Bandwidth of Pilot and Flight-Control System

Natural Linear-System Modes - Review

Ref. Sec. 10.1.1

Given the linear system (Physical inputs **u** and physical responses **y**)

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t), \qquad \mathbf{x} = \mathbf{M}\boldsymbol{\eta}, \qquad \mathbf{M}^{-1}\mathbf{A}\mathbf{M} = \boldsymbol{\Lambda} \tag{10.1}$$

Right and left eigenvectors

$$\mathbf{M} = \begin{bmatrix} \boldsymbol{\nu}_1 & \boldsymbol{\nu}_2 & \cdots & \boldsymbol{\nu}_n \end{bmatrix} \qquad \mathbf{M}^{-1} = \begin{bmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \\ \vdots \\ \boldsymbol{\mu}_n \end{bmatrix} \tag{10.4}$$

Then the dynamics of the natural modes are given by the decoupled eqns.

$$\dot{\boldsymbol{\eta}}(t) = \boldsymbol{\Lambda}\boldsymbol{\eta}(t) + \mathbf{M}^{-1}\mathbf{B}\mathbf{u}(t), \quad \Rightarrow \quad \dot{\eta}_i(t) = \lambda_i \eta_i(t) + \boldsymbol{\mu}_i \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{M}\boldsymbol{\eta}(t) + \mathbf{D}\mathbf{u}(t) \tag{10.8}$$

(Do not confuse w. vibration modal coordinates and mode shapes)

Now let $\mathbf{C} = \mathbf{I}$, $\mathbf{D} = \mathbf{0}$ ➔

$$\mathbf{y}(t) = \sum_{i=1}^{n} \boldsymbol{\nu}_i \eta_i(t)\mathbf{y} = \boldsymbol{\nu}_1\eta_1(t) + \boldsymbol{\nu}_2\eta_2(t) + \ldots + \boldsymbol{\nu}_n\eta_n(t) \tag{10.15}$$

Eigenvectors determine how each mode contributes to each physical response. And note that the j'th element of $\boldsymbol{\nu}_i$ will have the units of the physical response y_j.

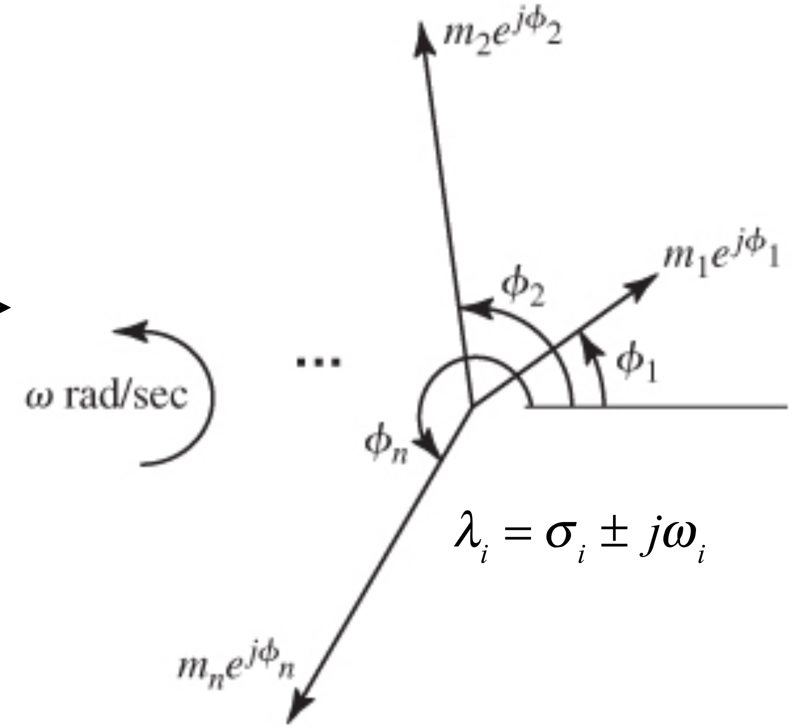
Each eigenvector therefore constitutes a mode shape, similar to the vibration case.

Eigenvectors, Phasor Diagrams and Modal Responses Ref. Sec. 10.1.1

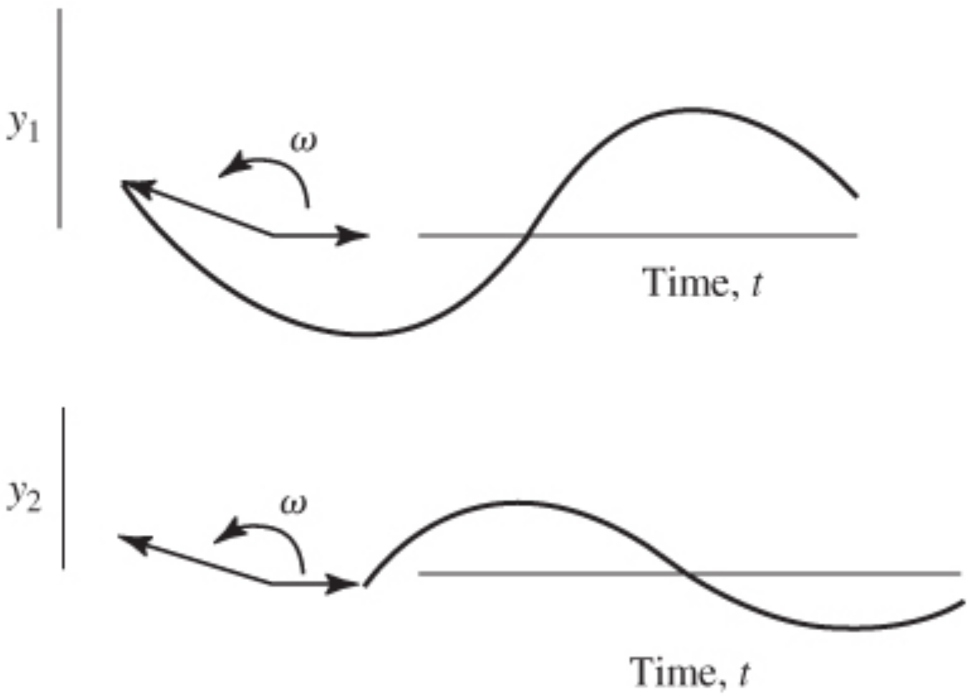
Eigenvector in Polar Form

$$\mathbf{v}_i = \begin{Bmatrix} m_1 e^{j\phi_1} \\ m_2 e^{j\phi_2} \\ \vdots \\ m_n e^{j\phi_n} \end{Bmatrix}$$

Phasor Diagram



$$\lambda_i = \sigma_i \pm j\omega_i$$



The generation of the pure modal time responses may be visualized as shown.

- Rate of decay determined by σ_i
- Frequency determined by ω_i
- Relative phasing determined by ϕ_j's
- Relative magnitudes determined by m_j's

Eigenvectors and Impulse Residues

Ref. Sec. 10.1.2

Next, consider an impulse response (transfer function), expanded in partial-fractions

$$y(s) = g(s) = \frac{R_1}{(s-\lambda_1)} + \cdots + \frac{R_n}{(s-\lambda_n)}, \qquad R_k = \left((s-\lambda_k)g(s)\right)\big|_{s=\lambda_k} \tag{10.21}$$

So the residue R_k also determines the contribution of mode i to the physical response.

Now recall the modal matrix $\mathbf{M}$

$$\mathbf{M} = \begin{bmatrix} \mathbf{v}_1 & \cdots & \mathbf{v}_n \end{bmatrix}, \qquad \mathbf{M}^{-1} = \begin{bmatrix} \boldsymbol{\mu}_1 \\ \vdots \\ \boldsymbol{\mu}_1 \end{bmatrix}$$

Right and left eigenvectors

(10.22)

(10.24)

And write the transfer-function matrix

$$\mathbf{TF}(s) = \left[\mathbf{CM} \left[diag\left(\frac{1}{s-\lambda_i} \right) \right] \mathbf{M}^{-1}\mathbf{B} \right] = \mathbf{C} \sum_{k=1}^{n} \frac{\left[\mathbf{v}_k \boldsymbol{\mu}_k \right]}{(s-\lambda_\kappa)} \mathbf{B}$$

So each transfer function may be expressed as

(10.25)

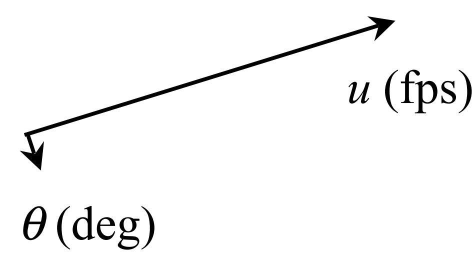
$$g_{i,j}(s) = \sum_{k=1}^{n} \frac{(\mathbf{c}_i \mathbf{v}_k)(\boldsymbol{\mu}_k \mathbf{b}_j)}{(s-\lambda_\kappa)} = \sum_{k=1}^{n} \frac{R_k}{(s-\lambda_\kappa)}$$

- Therefore, the left and right eigenvectors determine the residues.
- Pole-zero cancellation ➔ that pole's residue will be zero.
- Residue magnitudes indicate significance of modes in that response.

Modal Analysis - Longitudinal Axis
Large, High Speed Aircraft - Rigid Model

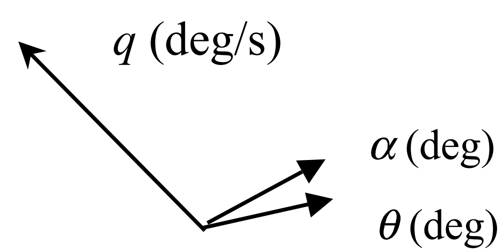
Eigenvalues | **Eigenvectors** | **Mode Shapes**

$$\lambda_{1,2} = 0.0002 \pm \ j0.066 \text{ /sec} \qquad \mathbf{v}_1 = \begin{Bmatrix} 0.993e^{j22.8^\circ} \ \text{(fps)} \\ 0.002e^{-j153.9^\circ} \ \text{(deg)} \\ 0.116e^{-j68.2^\circ} \ \text{(deg)} \\ 0.008e^{j21.7^\circ} \ \text{(deg)} \end{Bmatrix}$$



Classical Phugoid Mode

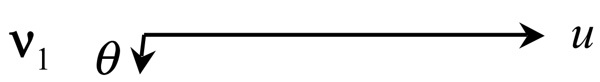
$$\lambda_{3,4} = -0.686 \pm \ j1.849 \text{ /sec} \qquad \mathbf{v}_3 = \begin{Bmatrix} 0.037e^{j71.5^\circ} \ \text{(fps)} \\ 0.439e^{j19.9^\circ} \ \text{(deg)} \\ 0.406e^{j7.4^\circ} \ \text{(deg)} \\ 0.800e^{j117.8^\circ} \ \text{(deg)} \end{Bmatrix}$$



Classical Short-Period Mode

State definition: $x^T = [u \text{ (fps)}, \alpha \text{ (deg)}, \theta \text{ (deg)}, q \text{ (deg/s)}]$

Modal Analysis of Flex Model
Large, High Speed Aircraft

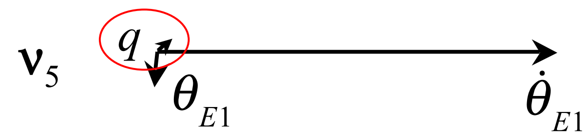


$$\lambda_{1,2} = 0.0002 \pm j0.066 \text{ /sec}$$

Phugoid

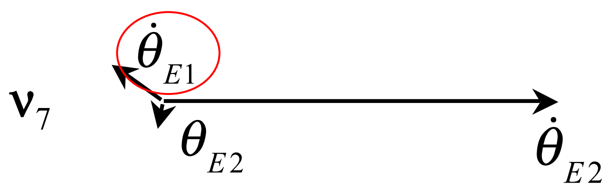
$$\lambda_{3,4} = -0.637 \pm j1.725 \text{ /sec}$$

"Short Period"
Note Elastic DOF Coupling



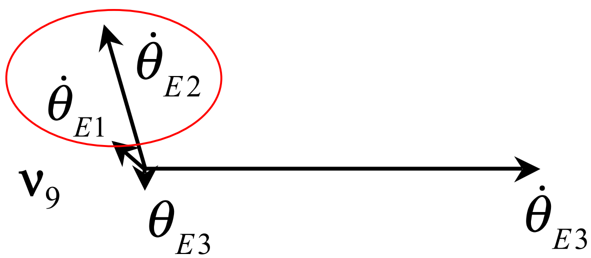
$$\lambda_{5,6} = -0.469 \pm j12.389 \text{ /sec}$$

"First Aeroelastic"
(1st Fuselage Bending)
Note RB DOF Coupling



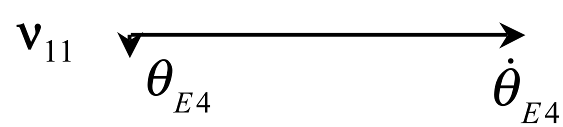
$$\lambda_{7,8} = -2.664 \pm j17.847 \text{ /sec}$$

"Second Aeroelastic"
(1st Wing Bending)
Note Elastic DOF Coupling



$$\lambda_{9,10} = -0.440 \pm j21.242 \text{ /sec}$$

"Third Aeroelastic"
(2nd Fuselage Bending)
Note Elastic DOF Coupling



$$\lambda_{11,12} = -0.427 \pm j22.045 \text{ /sec}$$

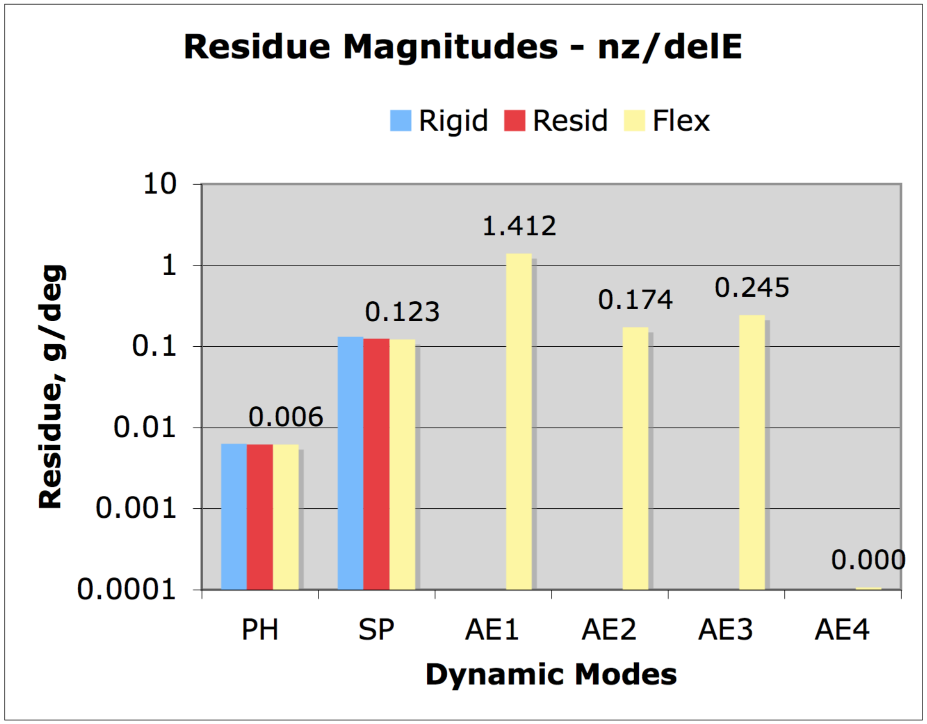
"Fourth Aeroelastic"
(3rd Fuselage Bending)

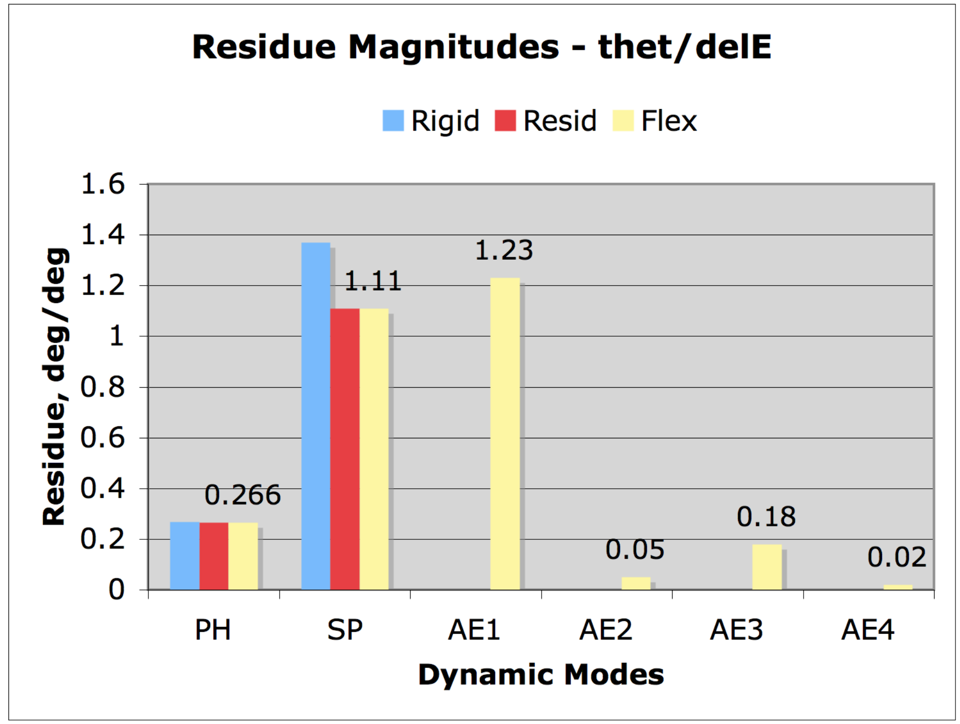
RB/Elastic Coupled Modes Now Exist (e.g., B2 Residual Pitch Oscillation)
Such Modes Are Not Consistent With Assumptions in HQ Database

State definition: $x^T = [u \text{ (fps)}, \alpha \text{ (deg)}, \theta \text{ (deg)}, q \text{ (deg/s)}, \theta_{CP\,Ei} \text{ (deg)}, \dot{\theta}_{CP\,Ei} \text{ (deg/s)}, i = 1 \ldots 4]$

Example - Impulse Residues
Large, High-Speed Aircraft

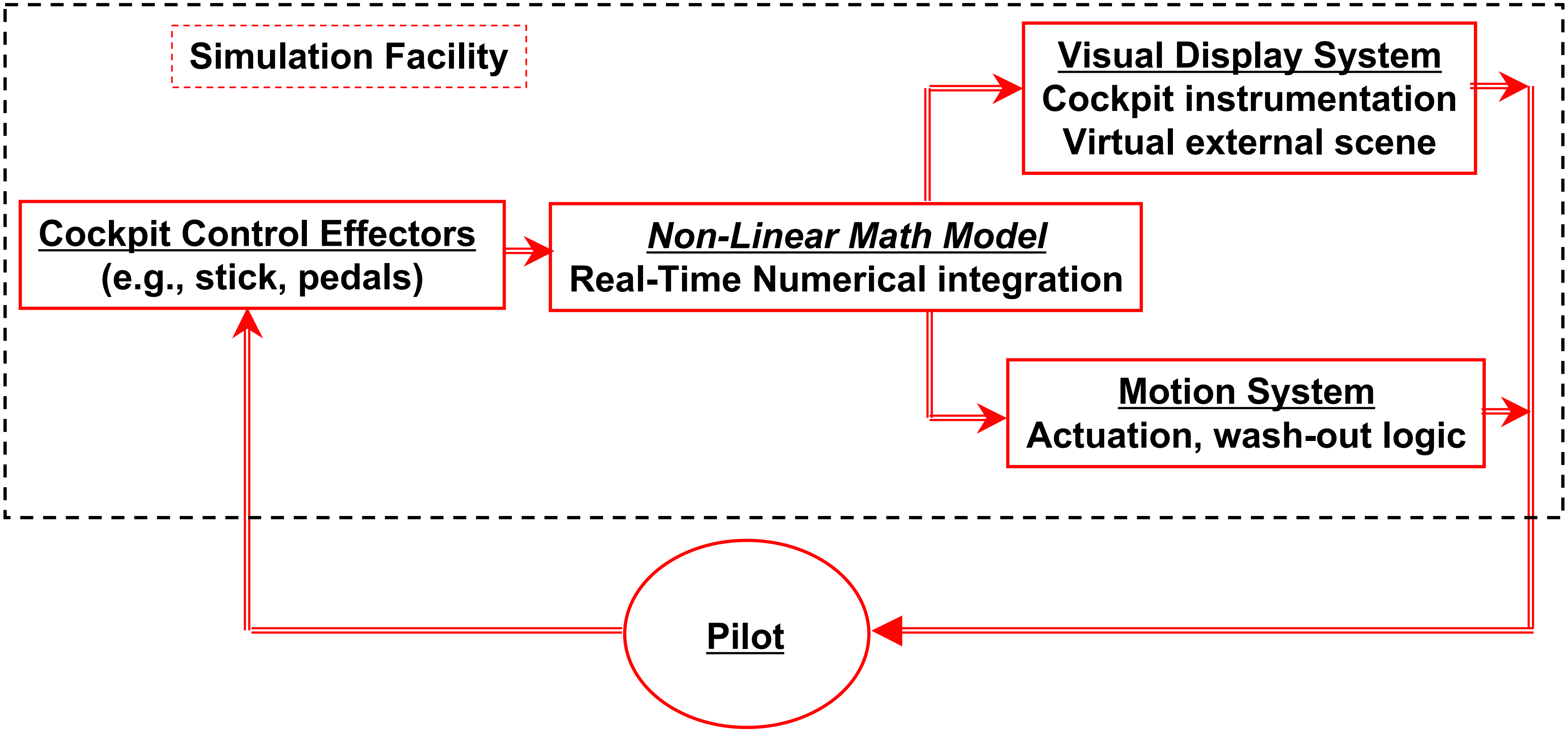
Vertical Acceleration and Pitch Attitude (Cockpit)





- First aeroelastic mode at least as significant as SP in these impulse responses

Components of a Piloted Real-Time Simulation



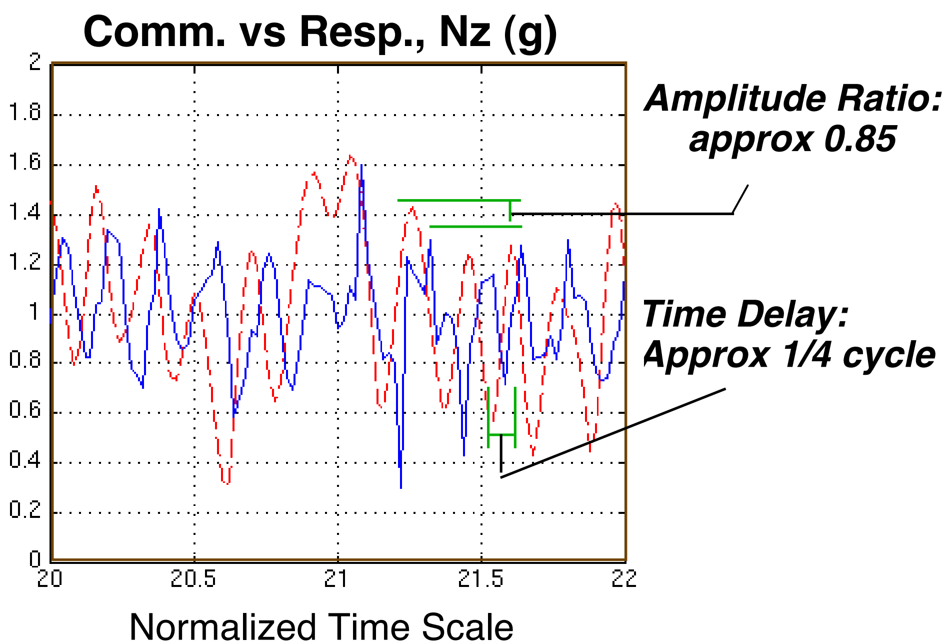
Component specs compatible with simulated system dynamics

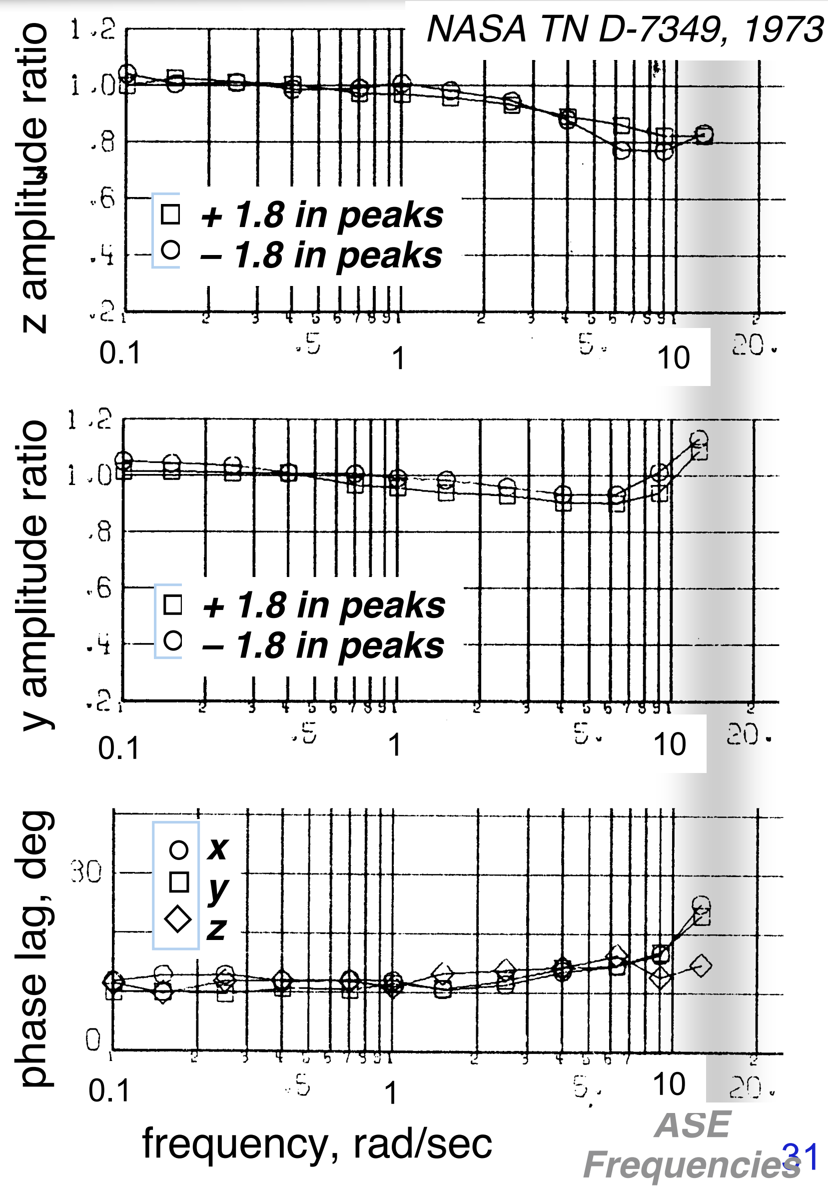
Simulation Considerations

Real-time requirement on
numerical integration

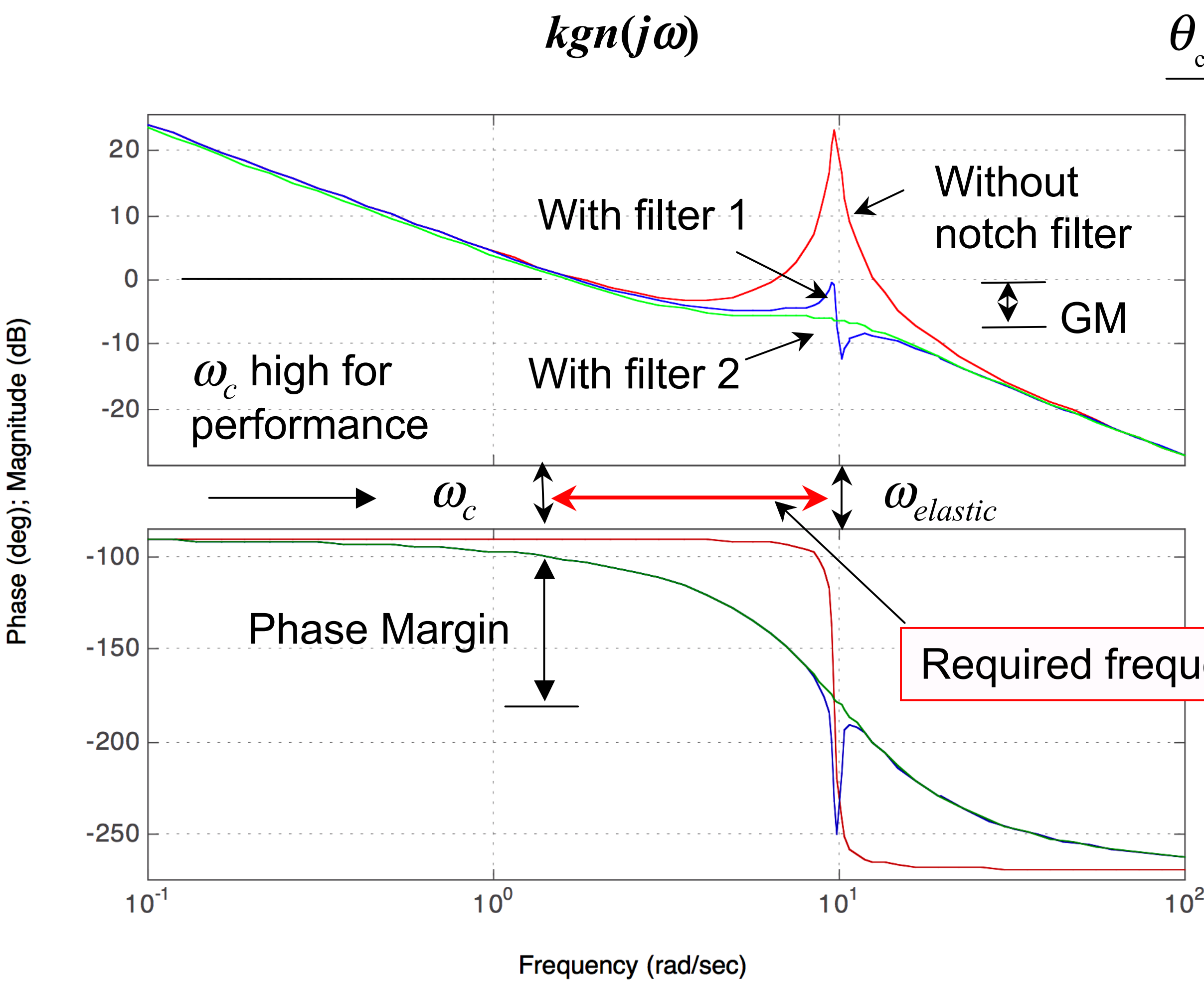
Motion-system limits
- Displacement limits
- Dynamic response limits

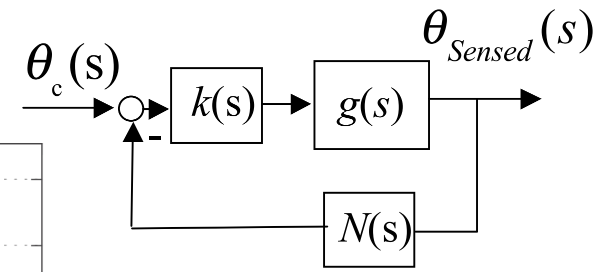
Simulation of Flex Dynamics Challenging





Example Flight-Control Issue
Effect of Notch Filter in Generic Control Loop





Filter 2 perfectly tuned

- Elastic Mode Limits Achievable Control-System Performance
- Notch Filter Must Be Properly Tuned to Aeroelastic Mode Frequency

Flight Control Issues Due to Flex Effects

- Flex dynamics can destabilize the flight-control system
- Flex dynamics introduces phase loss well below lowest frequency flex mode frequency (with notch or low-pass filtering)
- Flex effects limit achievable bandwidth (crossover frequencies) of flight-control system
- Sensor placement extremely important - depends on vibration mode shapes
- Flex effects increase complexity (cost) of flight-control systems (e.g., filters)
- Active structural-mode-control system may be required (e.g., B1, XB-70)

Summary & Conclusions

- Effects of flexibility on aircraft flight dynamics can be significant

  Handling and ride qualities
  Flight-control synthesis

- New vehicle configurations/requirements may amplify these effects
- Vibration-modal data and flex models required to support flight-control design
- Real-time simulation of elastic vehicles encounters new issues - sim limitations
- Renewed emphasis on cross-disciplinary modeling/analysis efforts needed
- Require math-model structure and methodology to be compatible with flight-dynamics applications
- Such an approach was outlined - many extensions possible
- Working across disciplines in new areas:

  Requires extra effort - must work hard to understand the other guy's problems
  Requires clarity in terminology, definitions, etc.