Probabilistic fatigue damage prognosis and uncertainty management

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Outline

- Problem Statement
- Background
- IVHM milestones being addressed
- Approach
- Results
- Conclusions
- Future Plans
Problem Statement

• **Physics-based probabilistic fatigue damage prognosis methodology**
  - Existing models are not suitable for concurrent prognosis and diagnosis analysis

• **Comprehensive uncertainty management framework for prognostic algorithms**
  - A comprehensive uncertainty quantification, propagation and updating scheme is lacking for prognostic algorithms

• **Rigorous model verification and validation methodology and its associated metrics**
  - No available prognosis metrics for time-dependent RUL prediction

• **Experimental testing to demonstrate, validate, and compare fatigue damage prognostic algorithms**
  - Multi-level experimental study for hypotheses verification, prediction validation, and application demonstration
Validation and uncertainty management of prognostic algorithms

- Small and long crack growth
- Multi-axial fatigue modeling
- Life prediction methodology
- Analytical and simulation methods

Probabilistic Fatigue Prognosis

- Random process theory
- Bayesian updating
- Usage monitoring and sensors
- Non-destructive inspection

Uncertainty Management

- Model error quantification
- Calibration under uncertainty
- Surrogate modeling
- Validation metrics and criteria

Model Validation

- Design of Experiments
- Uniaxial fatigue testing
- Multiaxial fatigue testing
- Hybrid simulation/experimental testing

Stochastic crack growth rate curve

Uncertainty quantification and propagation

Bayes network

Surrogate modeling
IVHM milestones being worked

- **IVHM 3.3.2**
  - Guidelines for fidelity of prognostic estimates, “…describes the appropriate level of fidelity for physics-based models for prognostics on subsystems and components.”

- **IVHM 3.3.3**
  - Methodology for assessing the performance of prognostic algorithms and methods, “…describes a rigorous statistical methodology for assessing the quality of prognostic algorithms.”

- **IVHM 3.3.5**
  - Assessment of the ability to perform prognostic reasoning for at least four of the adverse events listed in Table 2 (as specified in the RTIP) with performance improvements …

- **IVHM 1.2.3.7**
  - "Validated methodologies for prognostics uncertainty management and representation… shrink the uncertainty bounds of prediction of damage progression by 50% as measured from the initial prediction to the end of life".

This is a three year award and currently starting year 2.
A multi-scale approach for structural fatigue damage prognosis

- **Stress amplitude (MPa)**
- **Fatigue life (N)**
- **Delta K (da/dN)**
- **da/dN curve – Paris, 1960's**
- **SN curve – Wholer, 1860's**

**da/dt relationship at a smaller time scale**

**Length (m)**

- 1e-10
- 1e-9
- 1e-8
- 1e-6
- 1e-3
- 1e-2
- 1

**Time**

- 1e-15
- 1e-12
- 1e-9
- 1e-6
- 1 sec.
- days
- years

**Stress (SIF)**

**Δt**

**dt**

**Fatigue life (N)**

**Length (m)**

**2009 Aviation Safety Program Technical Conference**
Model development

Geometric relation

\[ da = \frac{ctg \theta}{2} d\delta = C d\delta \]  

(1)

Crack Tip Opening Displacement

\[ \delta = \frac{4K^2}{\pi E \sigma_y} = \lambda \sigma^2 a \]  

(\( \lambda = \frac{4}{E \sigma_y} \))  

(2)

Instantaneous crack growth rate

\[ \frac{1}{C\lambda a} \frac{da}{dt} = \frac{2\sigma}{1 - C\lambda \sigma^2} \frac{d\sigma}{dt} \]  

(3)

General formulation of the model

\[ \dot{a} = H(\dot{\sigma}) \cdot H(\sigma - \sigma_{ref}) \cdot \frac{2C\lambda}{1 - C\lambda \sigma^2} \cdot \dot{\sigma} \cdot a \]  

(4)

Hypotheses 1: crack growth is controlled by the interaction of forward and reversed plastic zone, which are influenced by crack closure

Hypotheses 2: crack growth in not uniformly distributed within one cycle and remains constant during majority of the loading history
In-situ fatigue testing under optical microscope and in SEM

Nikon metallurgical microscope

Controller and PC

Jeol 7400-F SEM

In-situ optical microscope fatigue testing

In-situ SEM fatigue testing
Forward and reversed plastic zone measurement

- In-situ optical microscope testing is used to measure the plastic zone size within one loading cycle
- Image correlation technique is used to estimate the crack tip strain field
- Crack closure hypothesis is verified for Al-7075-T6
- Ongoing work to include the crack blunting mechanism
High resolution crack tip deformation and growth observation

- Both crack deformation and growth can be observed
- Crack only grows during part of the loading path and not in the unloading path
- Ongoing work focuses on the imaging analysis (registration and mapping) and additional testing under different crack growth rates
Comparison with experimental data for model prediction
State-space model for concurrent structural-material fatigue prognosis

- Coupled hierarchical state-space model

Structural dynamics

\[ m \ddot{x} + n \dot{x} + kx = f(t) \]

Fatigue crack growth

\[
\frac{da}{dt} = H(\sigma)H(\sigma - \sigma_{ref}) \cdot \frac{2C\lambda}{1-C\lambda\sigma^2} \cdot \dot{\sigma} \cdot a
\]

\[
H(x) = \begin{cases} 
1, & \text{if } x > 0 \\
0, & \text{if } x \leq 0 
\end{cases}
\]

One DOF system
Structural damage prognosis integrating health and usage monitoring systems

Usage monitoring system

Risk assessment
Decision making

Health monitoring system

Fatigue damage prognosis

Structural dynamics

\[
\begin{bmatrix}
\dot{y}
\end{bmatrix}_{2N\times 1} = \begin{bmatrix}
0_{N\times N} \\
[K]_{N\times N} \\
[C]_{N\times N}
\end{bmatrix} \begin{bmatrix}
y
\end{bmatrix}
\]

\[
\sigma = E(y, y...)
\]

Incremental crack growth model

\[
\dot{a} = f(\sigma, \sigma, x_i)
\]

Healthy state features

\[
H = g(\sigma, \dot{\sigma}, a, K, C,....)
\]
Framework of the proposed uncertainty management methodology

- A sound uncertainty management methodology
  - Uncertainty Quantification (UQ)
  - Uncertainty Propagation (UP)
  - Uncertainty Updating (UU)
  - Risk Assessment (RA)
Uncertainty quantification

- Physical variability
  - Loading (multi-axial variable amplitude)
  - Material Properties

- Data uncertainty
  - Sparseness of data available to quantify material property statistics
  - Measurement uncertainty (final crack size)

- Model uncertainty/_errors
  - Finite element discretization error (Richardson extrapolation)
  - Gaussian process surrogate model prediction
  - Coefficients of selected crack growth model
  - Model form error terms

- Uncertainty in inspection
  - Crack detected $\rightarrow$ Use crack size and measurement error in inference
  - No crack detected $\rightarrow$ use POD (Probability of detection) in inference
Advanced surrogate modeling for uncertainty propagation

Basic idea: model the output $Y$ as a Gaussian process which is indexed by the inputs $x$.

Training: Given $m$ training points $x_1, ..., x_m$, with corresponding outputs $Y = [Y(x_1), ..., Y(x_m)]^T$, the joint distribution of $Y$ is defined by

$$Y \sim N_m[f^T(x)\beta, \lambda R]$$

- $f^T$ – $q$ basis functions for the trend -- linear or quadratic
- $\beta$ – coefficients of the regression trend
- $\lambda$ – process variance, $\lambda = \sigma^2$
- $R$ – $m$ by $m$ matrix of correlations among the training points
Suppose the following information is known:

- limit state function (Eq. (a))
- target reliability/confidence level (Eq. (b))

\[
\begin{align*}
(a) &: g(x, y) = 0 \\
(b) &: \|x\| = \beta
\end{align*}
\]

Inverse FORM (IFORM) is to find a solution of \( y \) to satisfy the above constraints

- vector \( x \) : random variables (e.g., material properties, load, structural geometries, etc.)
- vector \( y \) : index variables (e.g., time, coordinates, variables with small randomness, etc.)
Efficient probabilistic fatigue life prediction using IFORM

- No sampling required and suitable for both ground-based and on-line prognosis

- Directly calculate the RUL at a given confidence/reliability level

  - Limit state function

  \[
  g(A, a_i, N) = \log \left( \int_{a_i}^{a_c} \frac{1}{A b^R [\Delta K - \Delta K_{th}]^m} \, da \right) - \log(N)
  \]

  - Iterative calculation using Newton-Raphson method

  \[
  X_{k+1} = X_k - a_1 \left( \frac{\nabla_x g(x, N) \cdot x}{\|\nabla_x g(x, y)\|^2} \right) - g(x, N) + \beta_{target} \left( \frac{\nabla_x g(x, N)}{\|\nabla_x g(x, N)\|} \right)
  \]

  \[
  \frac{\partial g}{\partial N} = -\frac{1}{N}
  \]
An example for probabilistic life prediction – Al 7075

- Proposed IFORM method capture the trend and the scatter in the experimental data
- Give similar prediction accuracy compared to that of the direct Monte Carlo method
- IFORM is very efficient compared to the direct Monte Carlo method


Maximum Relative Entropy approach for uncertainty updating

- Uncertainty updating is a critical component for the overall uncertainty management
  - Update our belief using observations of the system response and reduce prognosis scatter band
- Classical Bayesian method is widely used
  - Difficult to handle moment data [1], e.g. $\langle \sqrt{\theta} \rangle$
- Maximum Relative Entropy (MRE) approach seeks the posterior under the moment constraints
  maximize $I(p : \mu) = -\int dx d\theta \cdot p(x, \theta) \log(p(x, \theta) / \mu(x, \theta))$
  under constraints $c_2 : \int dx d\theta \cdot p(x, \theta) g(\theta) = \langle g(\theta) \rangle = G$
- Posterior from MRE approach is a generalized Bayesian solution
  $p(\theta) \propto \mu(\theta) \cdot \mu(x' | \theta) \cdot e^{\beta \cdot g(\theta)}$

Rigorous model verification and validation using prognosis metrics

- Visual graphical comparison is useful but does not provide quantitative judgment of the investigated prognostic algorithms
- Classical metrics
  - Based on statistical analysis, a large number of samples are required
  - Difficult to describe the prognosis performance over time
- Prognostics-based metrics [1]
  - Designed to describe how well an algorithm improve over time
  - Not based on statistics, no sample required
  - 4 metrics: Prognostic Horizon (PH), $\alpha$-$\lambda$ accuracy, Relative Accuracy (RA), Convergence
- Demonstration using experimental testing data [2-3]
  - Experimental data: Al 2024-T3 in Virkler’s and McMaster’s dataset
  - Physics model: fatigue crack growth analysis
  - Probabilistic prognosis: MRE and Bayesian

Prognosis metrics – Virkler’s dataset

### Convergence

<table>
<thead>
<tr>
<th>Metric</th>
<th>MRE</th>
<th>Bayesian</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAPE</td>
<td>8.66</td>
<td>10.93</td>
</tr>
<tr>
<td>Average Bias (cycles)</td>
<td>10956.27</td>
<td>14051.92</td>
</tr>
<tr>
<td>STD(cycles)</td>
<td>7628.77</td>
<td>9115.78</td>
</tr>
<tr>
<td>MSE(cycle^2)</td>
<td>178.23 x 10^6</td>
<td>280.5 x 10^6</td>
</tr>
<tr>
<td>PH_{α=10%}</td>
<td>183283</td>
<td>169451</td>
</tr>
<tr>
<td>RA_{λ=0.4}</td>
<td>0.92</td>
<td>0.89</td>
</tr>
<tr>
<td>CRA_{λ=0.4}</td>
<td>0.89</td>
<td>0.87</td>
</tr>
<tr>
<td>Convergence (RA)</td>
<td>74365.72</td>
<td>77349.24</td>
</tr>
</tbody>
</table>
Prognosis confidence bounds estimation (Virkler’s dataset)

- Both MRE and Bayesian can narrow down the confidence bounds using additional observations
- Similar conclusions can be seen from the McMaster’s data
- Differences between MRE and Bayesian are case dependent, especially on the choice of prior distribution during the updating process
- Additional theoretical and experimental work are ongoing for new validation and new metrics development
Conclusions

- A general physics-based probabilistic fatigue damage prognosis methodology has been developed
- Novel small time scale fatigue formulation for concurrent multiscale fatigue damage modeling
- Comprehensive uncertainty quantification framework including various modeling and measurement errors
- Advanced surrogate modeling based on Gaussian Process (GP)
- Efficient probabilistic life prediction method for both ground-based and on-line prognosis
- Maximum Relative Entropy (MRE)/Bayesian updating to shrink the confidence bounds in the life prediction
- Rigorous prognostics-based metrics for quantitative algorithm performance evaluation
- Advanced in-situ optical and SEM testing for hypotheses validation
Next Steps

- Extend the developed fatigue modeling to general multiaxial random loading
- Develop a general computational methodology for the structural level fatigue prognosis based on the developed material model
- Develop new validation metrics for probabilistic prognostic algorithm comparison
- Global sensitivity analysis to investigate the effect of different uncertainty sources on prognosis
- Develop a general methodology to handle the uncertainty from unknown future loading and investigate its impact on the health management
- Extend the Bayesian framework for loading updating based on usage monitoring system
- Continue the experimental testing to supply validation data and support the model development