Probabilistic fatigue damage prognosis and uncertainty management

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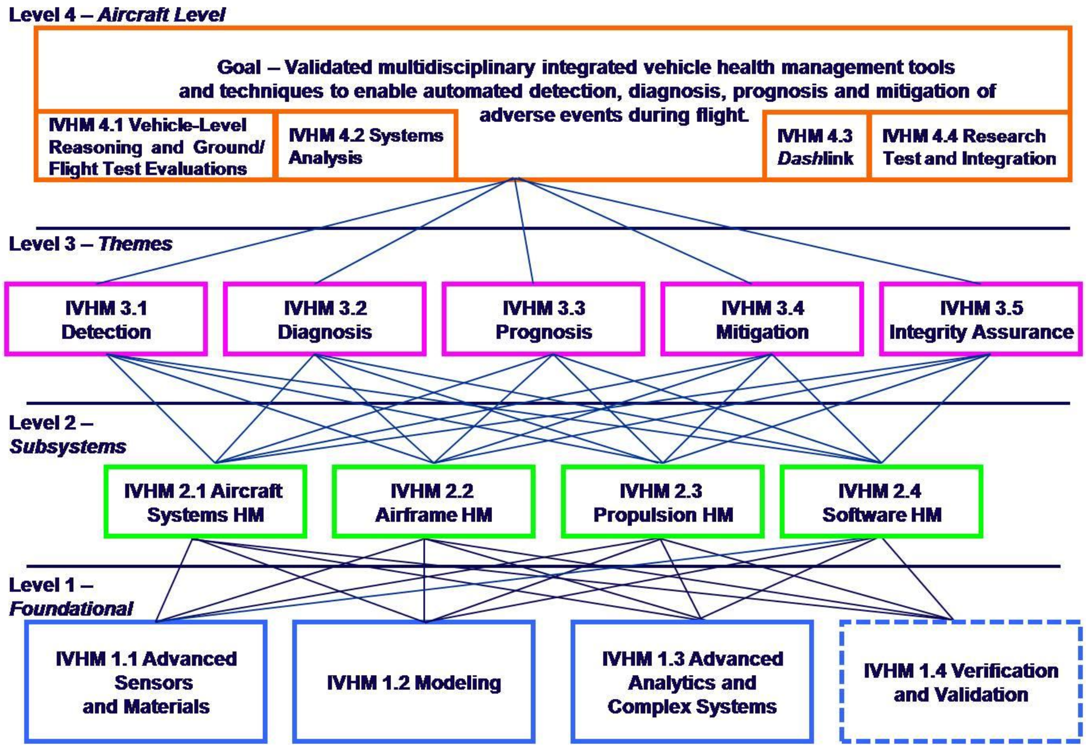
Outline

- Problem Statement
- Background
- IVHM milestones being addressed
- Approach
- Results
- Conclusions
- Future Plans



Problem Statement

- Physics-based probabilistic fatigue damage prognosis methodology

  - Existing models are not suitable for concurrent prognosis and diagnosis analysis

- Comprehensive uncertainty management framework for prognostic algorithms

  - A comprehensive uncertainty quantification, propagation and updating scheme is lacking for prognostic algorithms

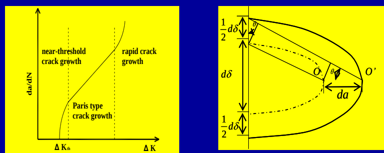
- Rigorous model verification and validation methodology and its associated metrics

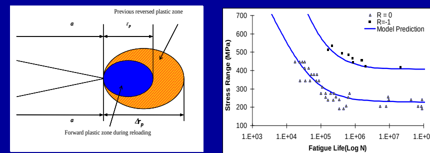
  - No available prognosis metrics for time-dependent RUL prediction

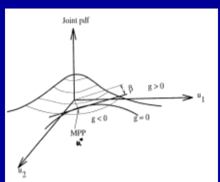
- Experimental testing to demonstrate, validate, and compare fatigue damage prognostic algorithms

  - Multi-level experimental study for hypotheses verification, prediction validation, and application demonstration

Validation and uncertainty management of prognostic algorithms

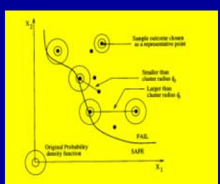

Fatigue crack growth modeling

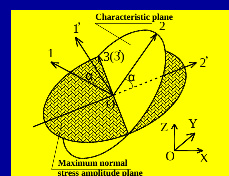

Life prediction methodology

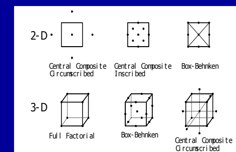

First-order reliability method

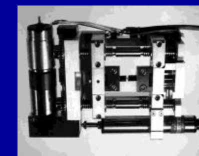
Probabilistic Fatigue Prognosis

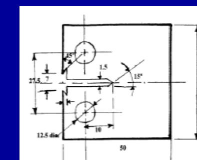
- Small and long crack growth
- Multi-axial fatigue modeling
- Life prediction methodology
- Analytical and simulation methods

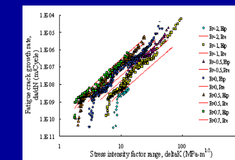

Adaptive importance sampling


Multi-axial fatigue analysis

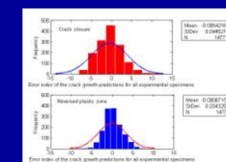

Design of Experiments

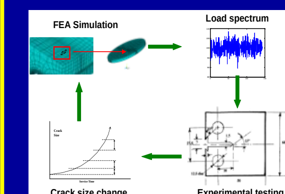

In-situ fatigue testing


Long crack testing


Crack growth measurements

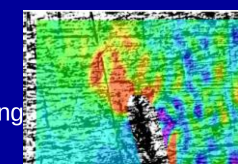
Experimental testing


Data analysis

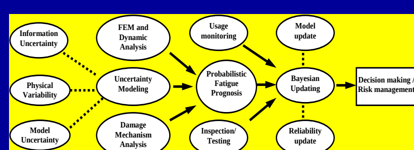


Hybrid simulation/experimental testing

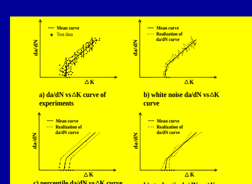
- Design of Experiments
- Uniaxial fatigue testing
- Multiaxial fatigue testing
- Hybrid simulation/experimental testing


Crack growth pattern and imaging analysis

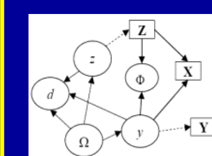
- Random process theory
- Bayesian updating
- Usage monitoring and sensors
- Non-destructive inspection

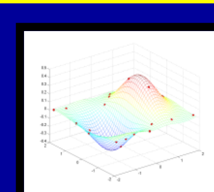


Uncertainty quantification and propagation

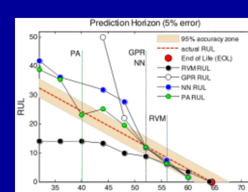

Stochastic crack growth rate curve

Uncertainty Management


Bayes network


Surrogate modeling

- Model error quantification
- Calibration under uncertainty
- Surrogate modeling
- Validation metrics and criteria



Validation metrics and criteria

Model Validation

IVHM milestones being worked

- **IVHM 3.3.2**

  - Guidelines for fidelity of prognostic estimates, "…describes the appropriate level of fidelity for physics-based models for prognostics on subsystems and components."

- **IVHM 3.3.3**

  - Methodology for assessing the performance of prognostic algorithms and methods, "…describes a rigorous statistical methodology for assessing the quality of prognostic algorithms."

- **IVHM 3.3.5**

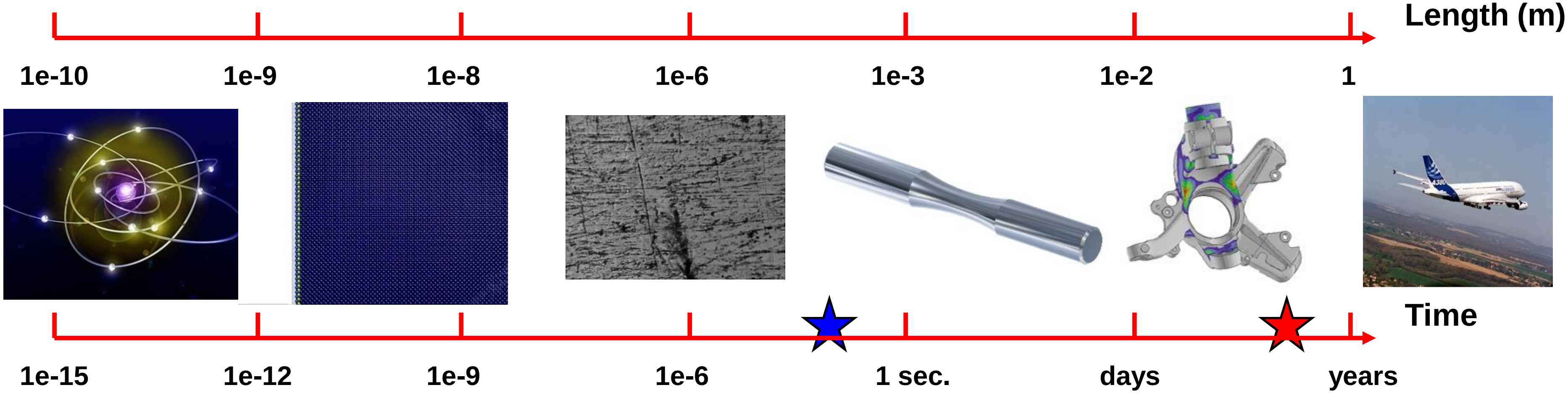
  - Assessment of the ability to perform prognostic reasoning for at least four of the adverse events listed in Table 2 (as specified in the RTIP) with performance improvements …

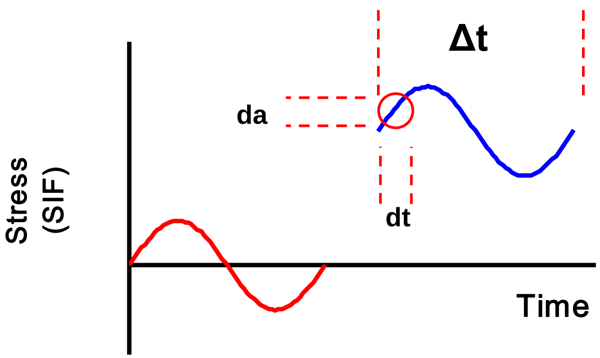
- **IVHM 1.2.3.7**

  - "Validated methodologies for prognostics uncertainty management and representation… shrink the uncertainty bounds of prediction of damage progression by 50% as measured from the initial prediction to the end of life".

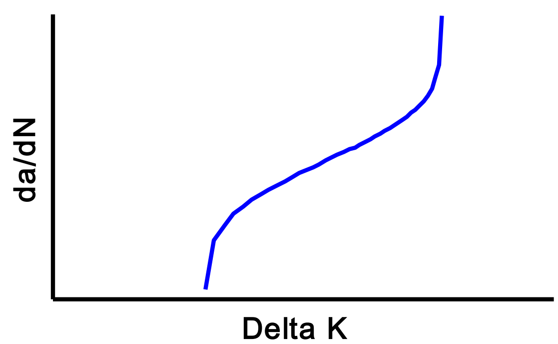
This is a three year award and currently starting year 2.

A multi-scale approach for structural fatigue damage prognosis

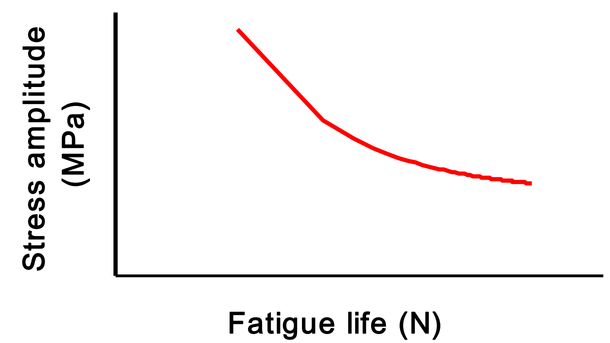




da/dt relationship at a smaller time scale



da/dN curve – Paris, 1960's



SN curve – Wholer, 1860's

Model development

Geometric relation

$$da = \frac{ctg\theta}{2} d\delta = C d\delta \qquad (1)$$

Crack Tip Opening Displacement

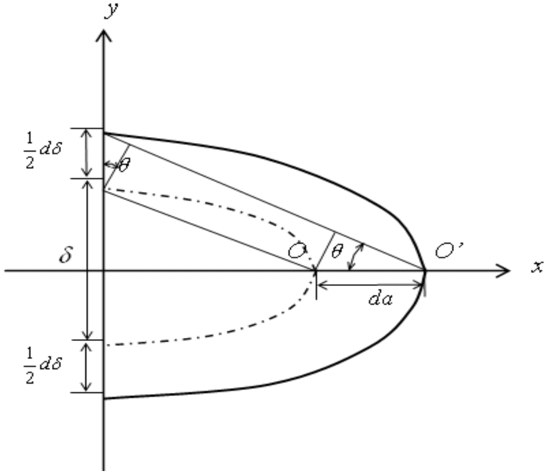
$$\delta = \frac{4K^2}{\pi E \sigma_y} = \lambda \sigma^2 a \qquad (\lambda = \frac{4}{E\sigma_y}) \qquad (2)$$

Instantaneous crack growth rate

$$\frac{1}{C\lambda a}\frac{da}{dt} = \frac{2\sigma}{1 - C\lambda\sigma^2}\frac{d\sigma}{dt} \qquad (3)$$

General formulation of the model

$$\dot{a} = H(\dot{\sigma}) \cdot H(\sigma - \sigma_{ref}) \cdot \frac{2C\lambda}{1 - C\lambda\sigma^2} \cdot \dot{\sigma} \cdot a \qquad (4)$$

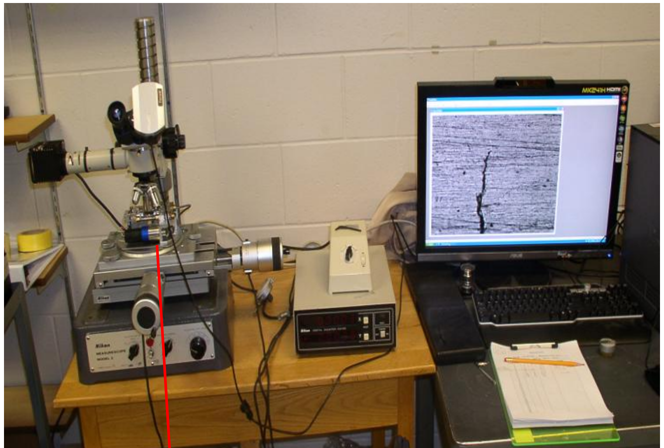


Schematic representation of crack tip geometry

Hypotheses 1: crack growth is controlled by the interaction of forward and reversed plastic zone, which are influenced by crack closure

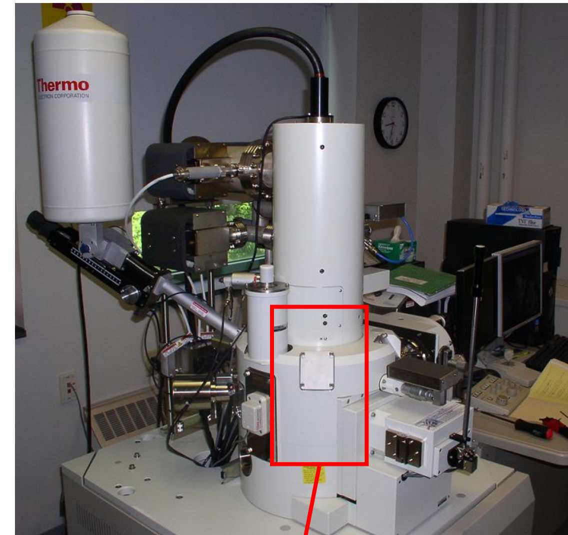
Hypotheses 2: crack growth in not uniformly distributed within one cycle and remains constant during majority of the loading history

In-situ fatigue testing under optical microscope and in SEM

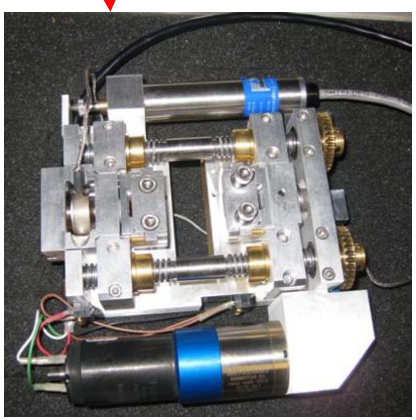


Nikon metallurgical microscope

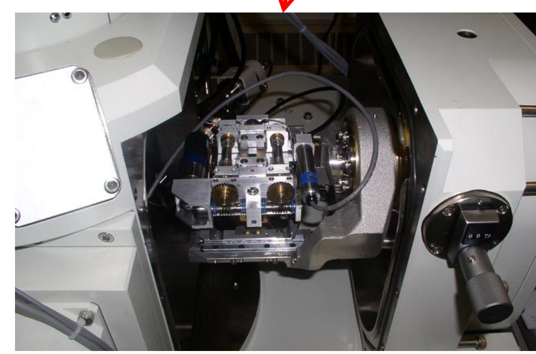
Controller and PC



Jeol 7400-F SEM

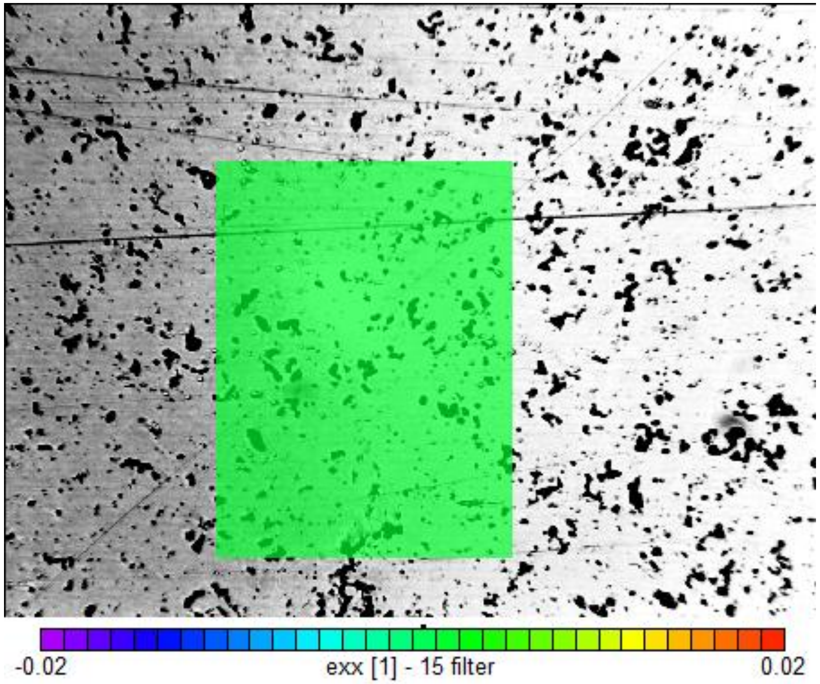


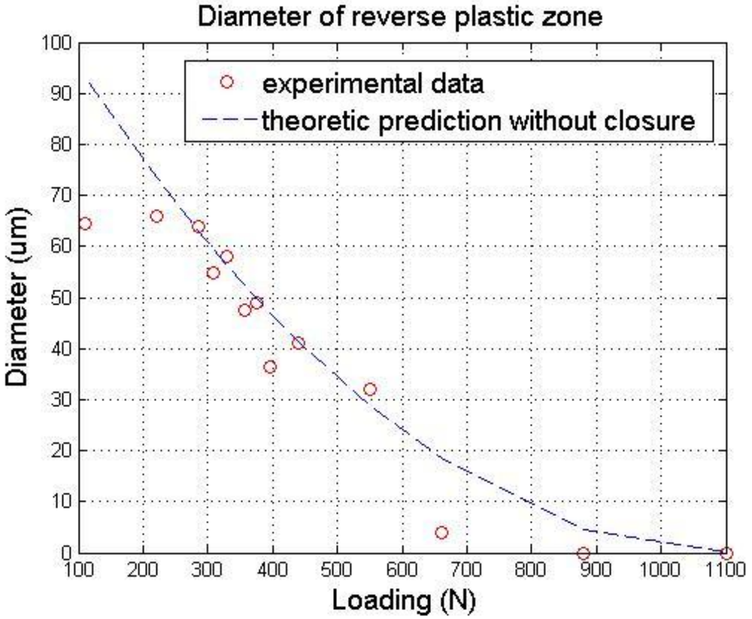
In-situ optical microscope fatigue testing



In-situ SEM fatigue testing

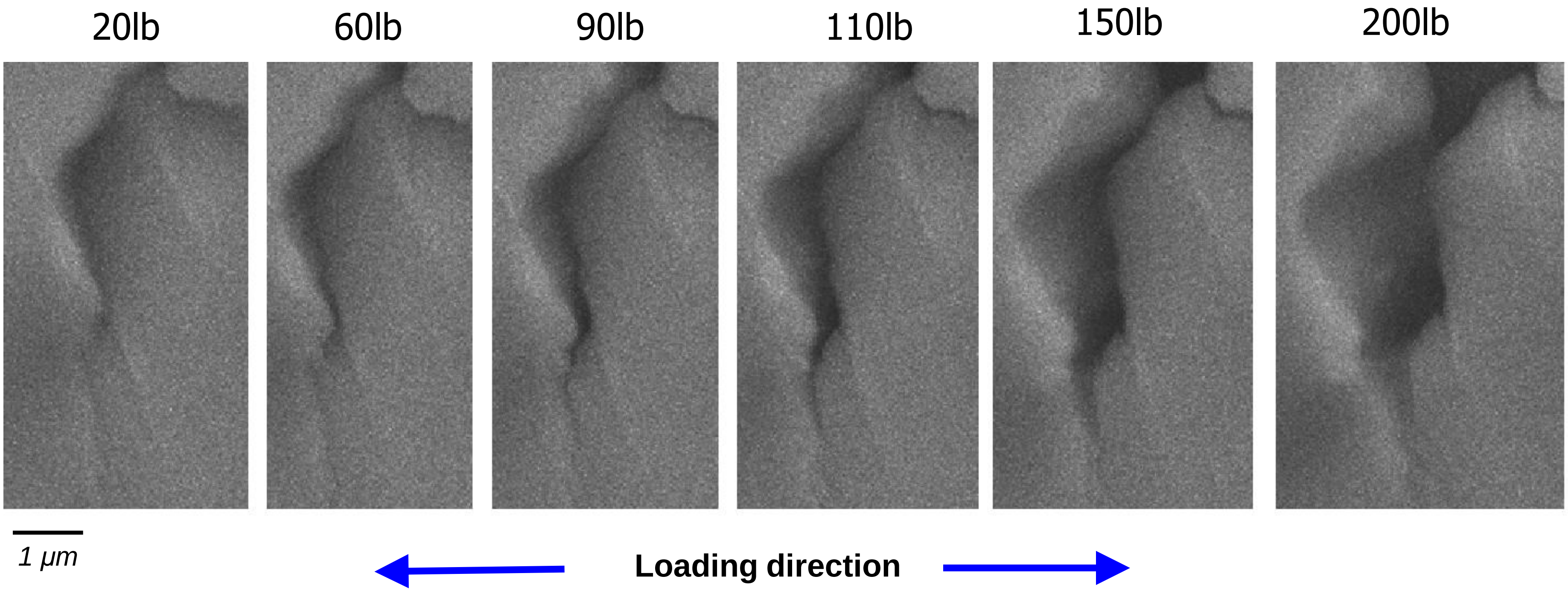
Forward and reversed plastic zone measurement





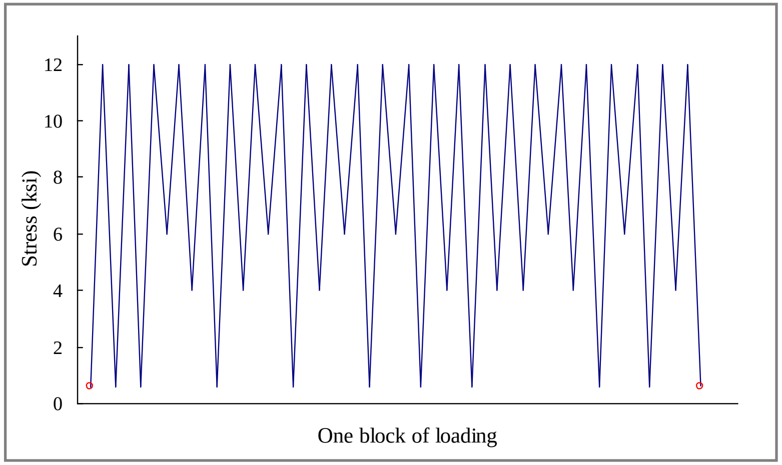
- In-situ optical microscope testing is used to measure the plastic zone size within one loading cycle
- Image correlation technique is used to estimate the crack tip strain field
- Crack closure hypothesis is verified for Al-7075-T6
- Ongoing work to include the crack blunting mechanism

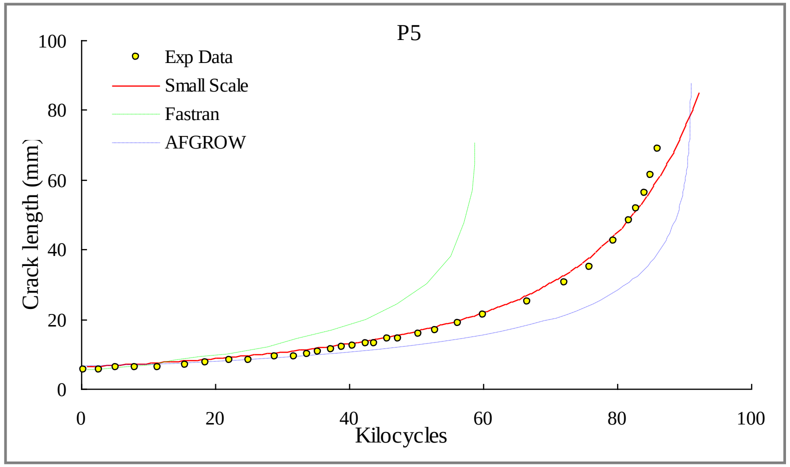
High resolution crack tip deformation and growth observation

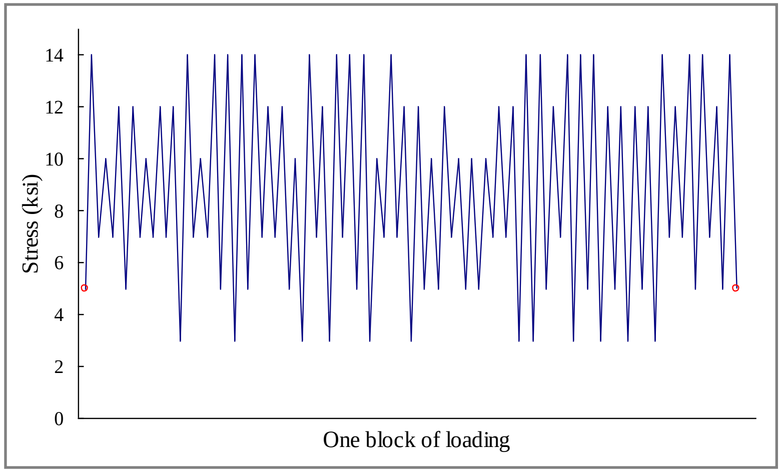


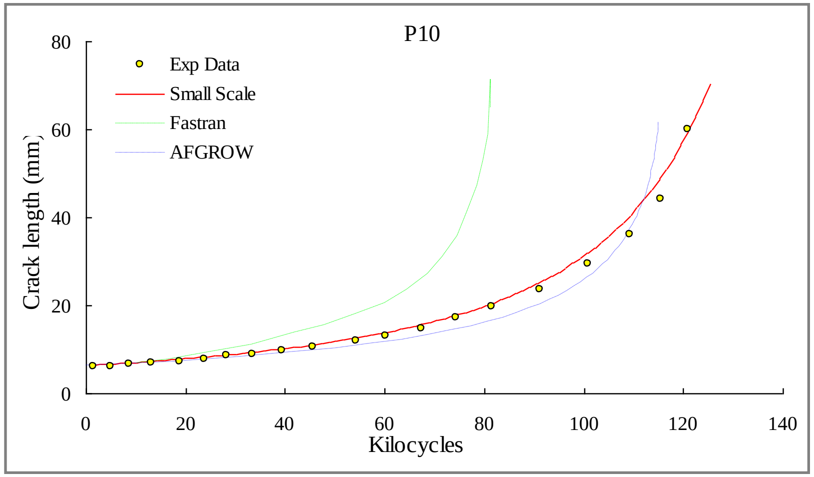
- Both crack deformation and growth can be observed
- Crack only grows during part of the loading path and not in the unloading path
- Ongoing work focuses on the imaging analysis (registration and mapping) and additional testing under different crack growth rates

Comparison with experimental data for model prediction









State-space model for concurrent structural-material fatigue prognosis

- Coupled hierarchical state-space model

Structural dynamics

$$m\ddot{x} + n\dot{x} + kx = f(t)$$

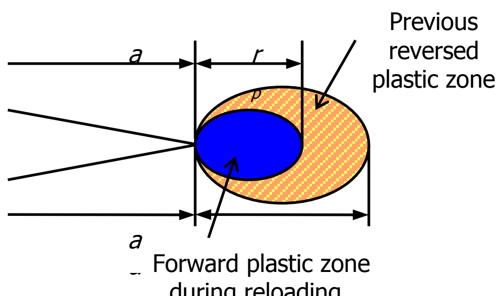
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (-k(x_3)/m)x_1 + (-n/m)x_2 + f(t)/m \\ \dot{x}_3 = H(g(x_2))H(g(x_1) - \sigma_{ref})\dfrac{2C\lambda}{1 - C\lambda g(x_1)^2} g(x_2)x_3 \end{cases}$$

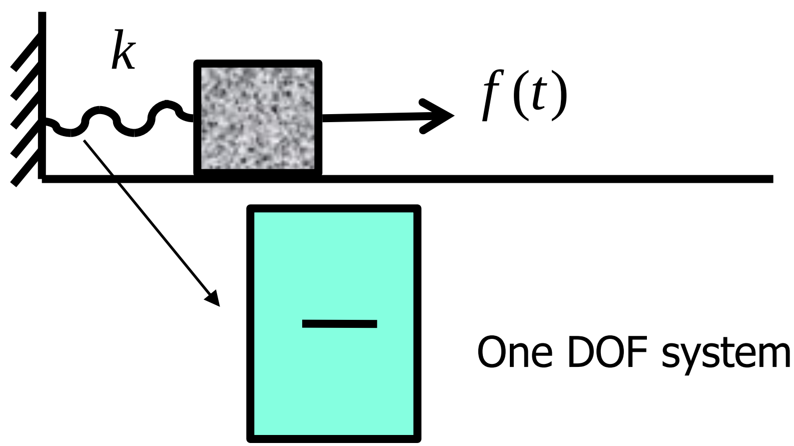
$$y = q(x_1, x_3)$$

Fatigue crack growth

$$\dot{a} = H(\dot{\sigma})H(\sigma - \sigma_{ref})\frac{2C\lambda}{1 - C\lambda\sigma^2}\dot{\sigma}a$$

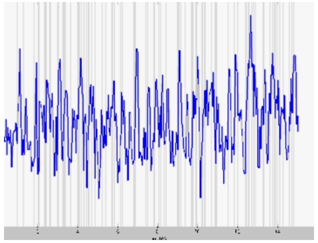
$$H(x) = \begin{cases} 1, & if \ x > 0 \\ 0, & if \ x \le 0 \end{cases}$$





Structural damage prognosis integrating health and usage monitoring systems

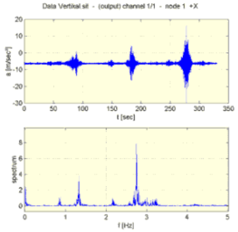
Usage monitoring system



Risk assessment Decision making

Fatigue damage prognosis

Health monitoring system



Structural dynamics

$$\begin{bmatrix} \dot{y} \end{bmatrix}_{2N\times 1} = \begin{bmatrix} [0]_{N\times N} & [I]_{N\times N} \\ [K]_{N\times N} & [C]_{N\times N} \end{bmatrix} [y]$$

$$\sigma = E(y, \dot{y} \ldots)$$

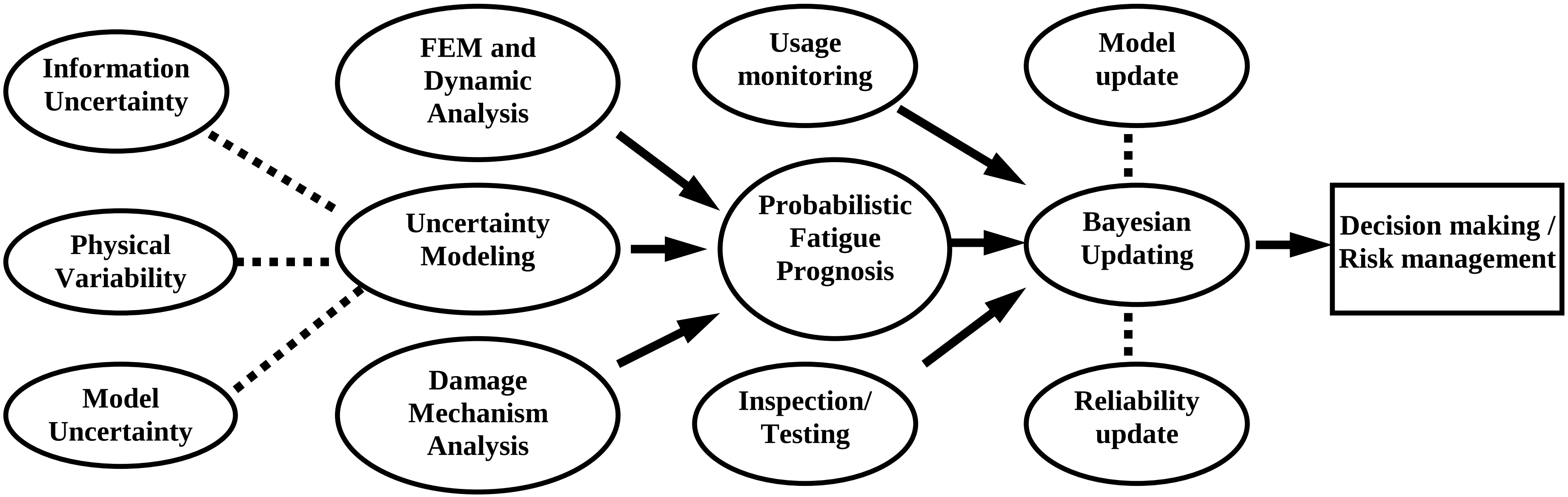
Incremental crack growth model

$$\dot{a} = f(\sigma, \dot{\sigma}, x_i)$$

Healthy state features

$$H = g(\sigma, \dot{\sigma}, a, K, C, \ldots.)$$

Framework of the proposed uncertainty management methodology



- A sound uncertainty management methodology
  - Uncertainty Quantification (UQ)
  - Uncertainty Propagation (UP)
  - Uncertainty Updating (UU)
  - Risk Assessment (RA)

Uncertainty quantification

- Physical variability
  - Loading (multi-axial variable amplitude)
  - Material Properties
- Data uncertainty
  - Sparseness of data available to quantify material property statistics
  - Measurement uncertainty (final crack size)
- Model uncertainty/errors
  - Finite element discretization error (Richardson extrapolation)
  - Gaussian process surrogate model prediction
  - Coefficients of selected crack growth model
  - Model form error terms
- Uncertainty in inspection
  - Crack detected → Use crack size and measurement error in inference
  - No crack detected → use POD (Probability of detection) in inference

Advanced surrogate modeling for uncertainty propagation

Basic idea: model the output Y as a Gaussian process which is indexed by the inputs $\mathbf{x}$.

Training: Given m training points $\mathbf{x}_1,\ldots,\mathbf{x}_m$, with corresponding outputs $\mathbf{Y}=[Y(\mathbf{x}_1),\ldots,Y(\mathbf{x}_m)]^T$, the joint distribution of Y is defined by

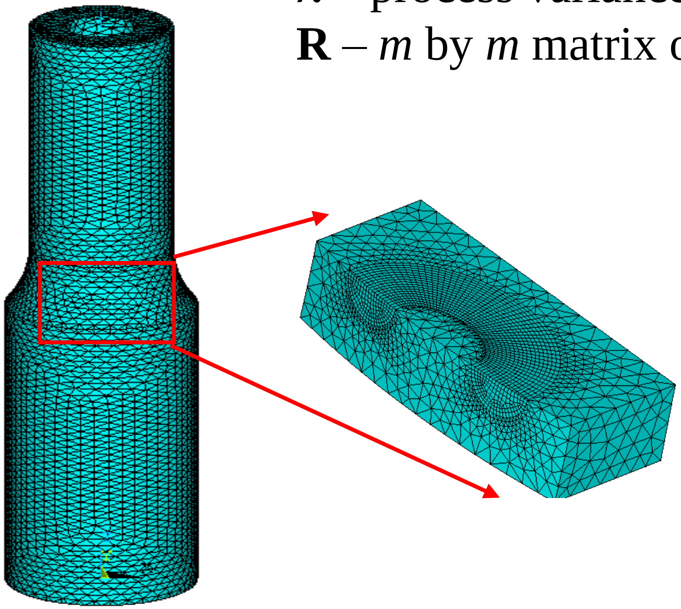
$$\mathbf{Y} \sim N_m[\mathbf{f}^T(\mathbf{x})\boldsymbol{\beta}, \lambda\mathbf{R}]$$

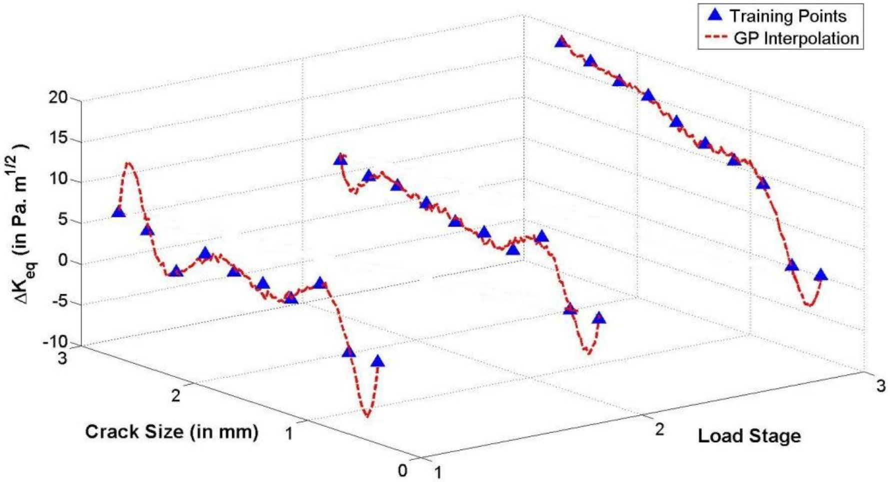
$\mathbf{f}^T$ – q basis functions for the trend -- linear or quadratic

β – coefficients of the regression trend

λ – process variance, $\lambda = \sigma^2$

$\mathbf{R}$ – m by m matrix of correlations among the training points





Suppose the following information is known:

limit state function (Eq. (a))

target reliability/confidence level (Eq. (b))

$$\begin{cases} (a): g(x, y) = 0 \\ (b): \|x\| = \beta \end{cases}$$

Inverse FORM (IFORM) is to find a solution of y to satisfy the above constraints

vector x : random variables (e.g., material properties, load, structural geometries, etc.)

vector y : index variables (e.g., time, coordinates, variables with small randomness, etc.)

Efficient probabilistic fatigue life prediction using IFORM

- No sampling required and suitable for both ground-based and on-line prognosis

- Directly calculate the RUL at a given confidence/reliability level

- Limit state function

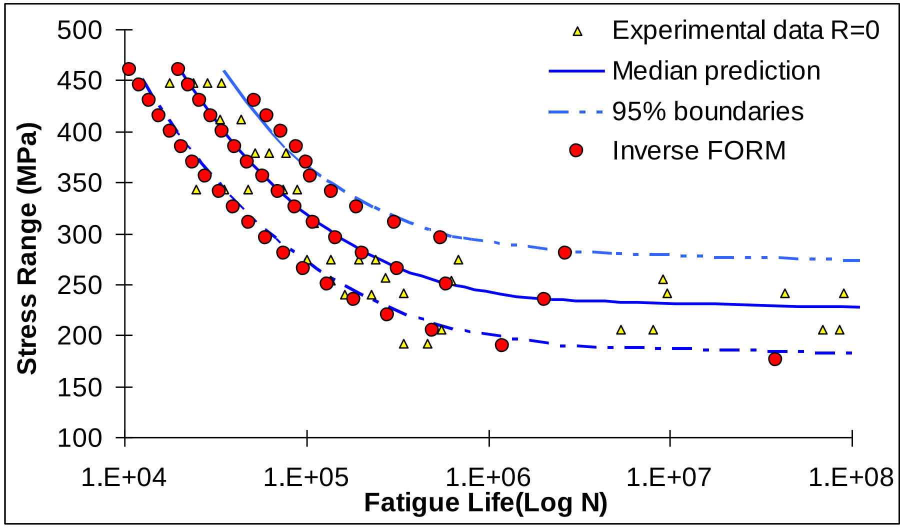
$$g(A, a_i, N) = \log\left(\int_{a_i}^{a_c} \frac{1}{Ab^R [\Delta K - \Delta K_{th}]^m} da \right) - \log(N)$$

$$\begin{cases} \nabla_x g(x) = \begin{cases} \dfrac{1}{A} \\ -\dfrac{\dfrac{1}{Ab^R [\Delta K - \Delta K_{th}]^m}}{\int_{a_i}^{a_c} \dfrac{1}{Ab^R [\Delta K - \Delta K_{th}]^m} da} \end{cases} \\ \dfrac{\partial g}{\partial N} = -\dfrac{1}{N} \end{cases}$$

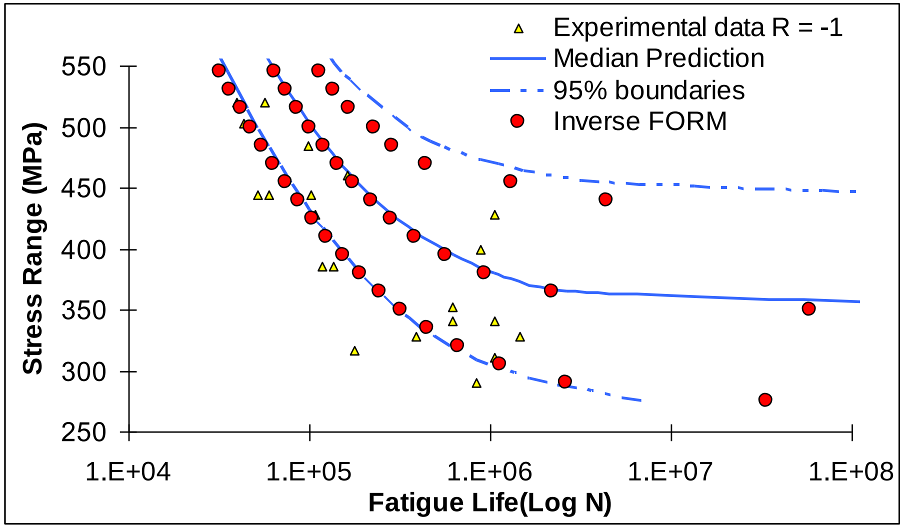
- Iterative calculation using Newton-Raphson method

$$\begin{Bmatrix} X_{k+1} \\ N_{k+1} \end{Bmatrix} = \begin{cases} X_k + a_1 \left(\dfrac{[\nabla_x g(x,N) \bullet x] - g(x,N)}{\|\nabla_x g(x,y)\|^2} \nabla_x g(x,N) - X_k \right) + a_2 \left(-X_k - \beta_{target} \dfrac{\nabla_x g(x,N)}{\|\nabla_x g(x,N)\|} \right) \\ N_k + a_2 \dfrac{[\nabla_x g(x,N) \bullet x] - g(x,N) + \beta_{target} \|\nabla_x g(x,N)\|}{\dfrac{\partial g(x,N)}{\partial N}} \end{cases}$$

An example for probabilistic life prediction – Al 7075



R=0



R=-1

- **Proposed IFORM method capture the trend and the scatter in the experimental data**
- **Give similar prediction accuracy compared to that of the direct Monte Carlo method**
- **IFORM is very efficient compared to the direct Monte Carlo method**

Liu, Y., Mahadevan, S., "Probabilistic fatigue life prediction using an equivalent initial flaw size distribution", International Journal of Fatigue, Vol. 31, Issue 3, pp. 476-487, 2009.

Xiang, Y., Lu, Z., **Liu, Y.**, "Crack growth-based fatigue life prediction using an equivalent initial flaw model. Part I: Uniaxial loading", International Journal of Fatigue, 2009. (in press)

Maximum Relative Entropy approach for uncertainty updating

- Uncertainty updating is a critical component for the overall uncertainty management

  - Update our belief using observations of the system response and reduce prognosis scatter band

- Classical Bayesian method is widely used $p(\theta) \propto \mu(\theta) \cdot \mu(x' \mid \theta)$
  - Difficult to handle moment data [1], e.g. $\langle \sqrt{\theta} \rangle$
- Maximum Relative Entropy (MRE) approach seeks the posterior under the moment constraints

  maximize $I(p:\mu) = -\int dx d\theta \cdot p(x,\theta) \log\left(p(x,\theta)/\mu(x,\theta)\right)$

  under constraints $c_2 : \int dx d\theta \cdot p(x,\theta) g(\theta) = \langle g(\theta) \rangle = G$

- Posterior from MRE approach is a generalized Bayesian solution

$$p(\theta) \propto \mu(\theta) \cdot \mu(x' \mid \theta) \cdot e^{\beta \cdot g(\theta)}$$

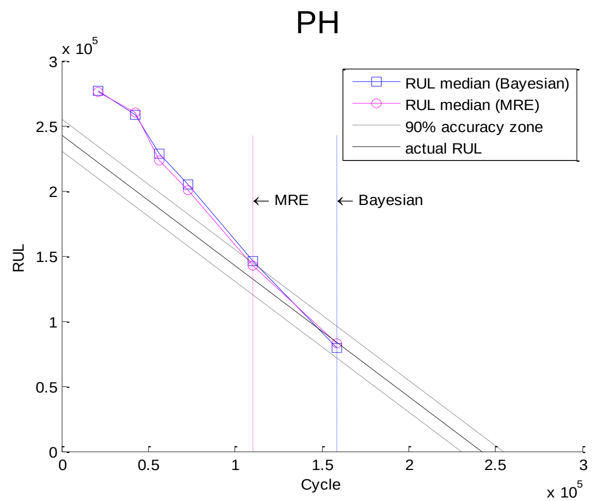
[1] A. Giffin and A. Caticha (2007). Updating probabilities with data and moments. In K. Knuth (Ed.), Bayesian Inference and Maximum Entropy Methods in Science and Engineering, AIP Conference Proceedings, 954:74

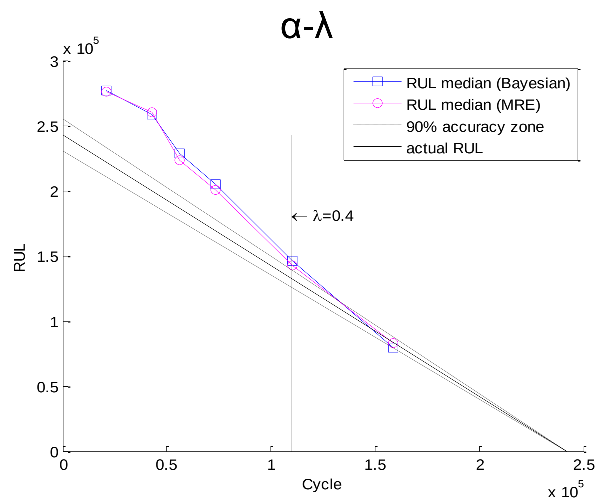
Rigorous model verification and validation using prognosis metrics

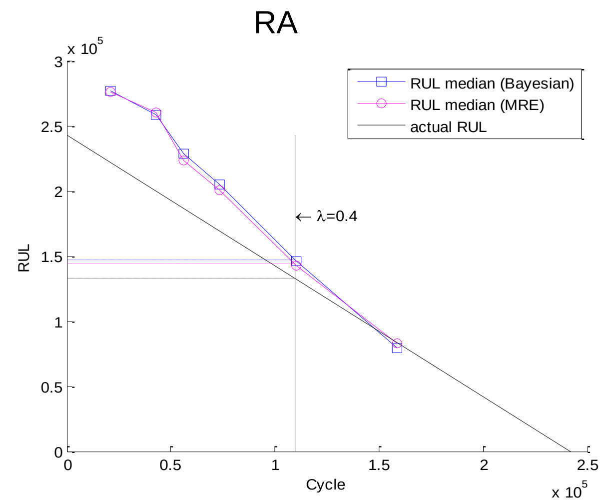
- Visual graphical comparison is useful but does not provide quantitative judgment of the investigated prognostic algorithms
- Classical metrics
  - Based on statistical analysis, a large number of samples are required
  - Difficult to describe the prognosis performance over time
- Prognostics-based metrics [1]
  - Designed to describe how well an algorithm improve over time
  - Not based on statistics, no sample required
  - 4 metrics: Prognostic Horizon (PH), *α-λ* accuracy, Relative Accuracy (RA), Convergence
- Demonstration using experimental testing data [2-3]
  - Experimental data: Al 2024-T3 in Virkler's and McMaster's dataset
  - Physics model: fatigue crack growth analysis
  - Probabilistic prognosis: MRE and Bayesian

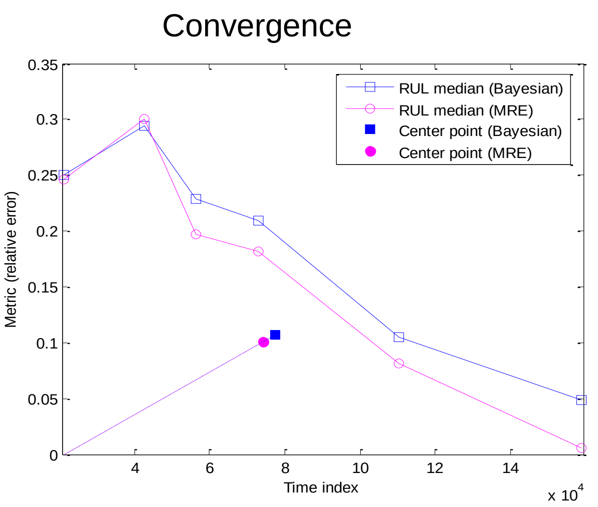
[1] A. Saxena, J. Celaya, B. Saha, S. Saha, and K. Goebel (2009). Evaluating algorithm performance metrics tailored for prognostics. IEEE Aerospace conference, 7-14 March 2009, pp. 1-13.
[2] Guan, X., Jha, R., **Liu, Y.**, "Probabilistic fatigue damage prognosis using maximum entropy approach", Journal of Intelligent Manufacturing, 2009. (accepted)
[3] Guan, X., **Liu, Y.**, Saxena, A., Celaya, J., Goebel, K., "Entropy-based probabilistic fatigue damage prognosis and algorithmic performance comparison", annual conference of the prognostics and health management society, San Diego, CA, 2009.

Prognosis metrics – Virkler's dataset



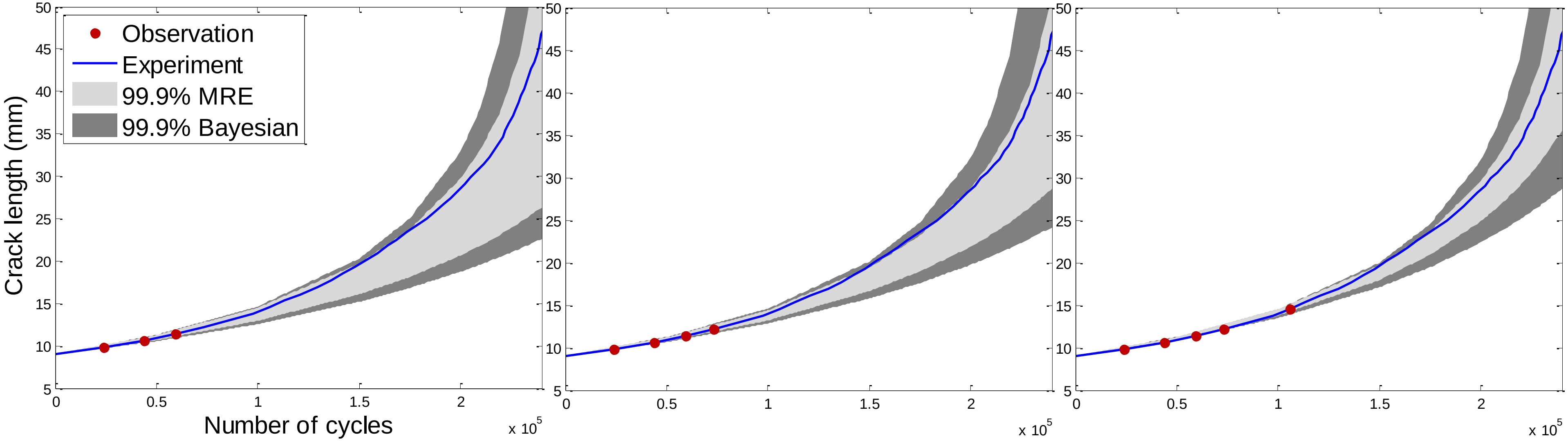






Metric	MRE	Bayesian
MAPE	8.66	10.93
Average Bias (cycles)	10956.27	14051.92
STD(cycles)	7628.77	9115.78
MSE(cycle^2)	178.23 x 10^6	280.5 x 10^6
$PH_{\alpha=10\%}$	183283	169451
RA $_{\lambda=0.4}$	0.92	0.89
CRA $_{\lambda=0.4}$	0.89	0.87
Convergence (RA)	74365.72	77349.24

Prognosis confidence bounds estimation (Virkler's dataset)



- Both MRE and Bayesian can narrow down the confidence bounds using additional observations
- Similar conclusions can be seen from the McMaster's data
- Differences between MRE and Bayesian are case dependent, especially on the choice of prior distribution during the updating process
- Additional theoretical and experimental work are ongoing for new validation and new metrics development

Conclusions

- A general physics-based probabilistic fatigue damage prognosis methodology has been developed
- Novel small time scale fatigue formulation for concurrent multiscale fatigue damage modeling
- Comprehensive uncertainty quantification framework including various modeling and measurement errors
- Advanced surrogate modeling based on Gaussian Process (GP)
- Efficient probabilistic life prediction method for both ground-based and on-line prognosis
- Maximum Relative Entropy (MRE)/Bayesian updating to shrink the confidence bounds in the life prediction
- Rigorous prognostics-based metrics for quantitative algorithm performance evaluation
- Advanced in-situ optical and SEM testing for hypotheses validation

Next Steps

- Extend the developed fatigue modeling to general multiaxial random loading
- Develop a general computational methodology for the structural level fatigue prognosis based on the developed material model
- Develop new validation metrics for probabilistic prognostic algorithm comparison
- Global sensitivity analysis to investigate the effect of different uncertainty sources on prognosis
- Develop a general methodology to handle the uncertainty from unknown future loading and investigate its impact on the health management
- Extend the Bayesian framework for loading updating based on usage monitoring system
- Continue the experimental testing to supply validation data and support the model development