

# 3D CONFLICT AVOIDANCE UNDER UNCERTAINTIES

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## Abstract

This paper presents a 3D conflict avoidance algorithm in the presence of uncertainty. The objective of the algorithm is to ensure the safety of the airspace in the event of a failure in the communication, navigation or surveillance systems. The algorithm minimizes the number of maneuvers required to maintain the safety of the airspace under degraded conditions. Uncertainties are modeled as an increase in the required separation distance between aircraft. A single maneuver for each aircraft is chosen to maintain safe separation. Maneuvers include heading change, speed change and flight level change. Maneuvers are simple to execute and guarantee a conflict-free configuration after execution. Their feasibility is constrained by weather avoidance, sector boundaries and aircraft performance. A Mixed Integer Program is used to determine the set of maneuvers to be executed.

## Notations

$S$	Set of all aircraft in the airspace.
$\mathcal{M}$	Set of potential maneuvers (speed changes, heading changes and flight level changes).
$\mathcal{M}_i^{feas}$	Set of feasible maneuvers for aircraft $i$ .
$\mathcal{K}_{ij}$	Set of combination of feasible maneuvers leading to no conflict between aircraft $i$ and $j$ .
$\mathcal{C}$	Set of clusters.
$\mathcal{W}$	Set of severe weather areas to be avoided.
$c_i^{m_k}$	Overall cost of aircraft $i$ executing maneuver $k$ .
$c_{ij}^{m_k}$	Cost of aircraft $i$ executing maneuver $k$ to avoid aircraft $j$ .
$C_p$	Cluster $p$ .
$m_i^k$	Binary variable indicating that aircraft $i$ executes maneuver $k$ .
$r_0$	Initial radius of avoidance.
$r_f$	Final radius of avoidance.
$\dot{r}$	Rate of growth of the radius of avoidance.
$t_c$	Time horizon for conflict detection.
$t_g$	Growth time for $r_0$ to $r_f$ for the radius of avoidance.
$t_s$	Time horizon for sector boundary crossing.
$t_w$	Time horizon for weather avoidance.

## Introduction

The new air traffic management concepts of operations such as described in NextGen[1] and SESAR[2] require significant advancements in Communication, Navigation and Surveillance (CNS) technologies. Such concepts of operation include high density operations where aircraft would fly close to each other, relying on accurate positioning systems and automation. If one of these systems fails, a backup system is needed to ensure the safety of the airspace. This process is called “graceful degradation”[3]. To ensure the safety of the airspace, there exists a minimum separation distance between aircraft that should be maintained at all times. The minimum separation distance between aircraft results, in what we call a circle of avoidance of radius  $r_0$  surrounding each aircraft for the horizontal plan. In the vertical dimension, the minimum separation is given by the flight levels. We say that two aircraft are in conflict or do not maintain the safety of the airspace if their circles of avoidance will intersect when no avoidance maneuver is taken. Literature on conflict detection and avoidance includes but is not limited to [4, 5, 6, 7] and many works include conflict detection and avoidance under uncertainties [8, 9]. Nevertheless, none of these algorithms deal with the problem of avoidance under degraded performance. In [3], we introduced a 2D avoidance algorithm in the presence of uncertainties. Uncertainties were used to model a degradation of performance in the CNS system. They are represented as an increase in the radius of the circle of avoidance surrounding each aircraft. The formulation presented in [3] is based on a Mixed Integer Linear Program and requires strong hypotheses such as the same speed for all aircraft. In this paper, we develop a formulation of the problem where 3D maneuvers are acceptable. The major changes from the algorithm presented in [3] are:

- 3D maneuvers including heading change, speed change, flight level change are authorized.
- Incorporation of constraints such as sector boundaries, weather avoidance and true aircraft performances.
- Aircraft can have different speeds.

The objective of this algorithm is to provide controllers with a simple set of avoidance maneuvers in the event of a loss of performance in the CNS system. The goal is not to optimize all avoidance maneuvers, but rather to focus on airspace safety and to be able to handle a large number of aircraft simultaneously. A set of maneuvers that solves all the potential conflicts in the case of growing radii of avoidance is first determined and then, a single maneuver for each aircraft is chosen by formulating and solving a mixed integer program. The cost of each maneuver depends on the imminence of the conflict. The paper is organized as follow: The algorithm used to compute all the feasible solutions is presented in the first section. The second section introduces the mixed integer programming (MIP) formulation used to choose among all the feasible solutions. The third section presents simulation results before the conclusion of the paper.

## Algorithm Overview

Consider a system of  $n$  aircraft in a 3-dimensional Cartesian airspace. The flat earth approximation is valid for the distances considered in this paper. The set of all aircraft in the airspace is denoted by  $\mathcal{S}$ . The horizontal position of aircraft  $i$  is  $(x_i, y_i)$  and its vertical position is given by its flight level  $FL_i$ . Its speed and heading are  $V_i$  and  $\theta_i$ , respectively. The algorithm is intended for en route traffic.

We first give an overview of the algorithm and then detail the different steps:

1. Organize aircraft by flight level.
2. Determine, for each aircraft  $i$ , the set of feasible maneuvers  $\mathcal{M}_i^{feas}$  (performance, weather, sector boundary, flight level change constraints).
3. Determine, for each pair of aircraft  $(i, j)$ , the set  $\mathcal{K}_{ij}$  of combinations of feasible maneuvers  $K_{ij}^{kl}$  that do not lead to a conflict.
4. Cluster aircraft to reduce computing time.
5. Find the set of maneuvers (one per aircraft) that minimizes the overall cost and ensures the safety of the airspace.

## Organization of Aircraft by Flight Level

During the en-route phase of the flight, aircraft maintain constant pressure altitude, corresponding to flight levels. Pilots need to obtain a clearance to increase altitude when the weight of the aircraft permits. Then,

they can climb to the next allowed flight level. During en-route phases, flight level changes do not occur very often. It is then reasonable to split the overall airspace into layers corresponding to flight levels. Aircraft can transfer from one layer to another but the movement of each aircraft is considered two-dimensional between flight level change phases.

## Determination of Feasible Maneuvers

To provide controllers with a simple set of avoidance maneuvers, the set  $\mathcal{M}$  of potential maneuvers for all the aircraft includes the following: No change, speed changes ( $\pm\Delta V$ ), heading changes ( $\pm\Delta\theta$ ) and flight level changes ( $\pm\Delta FL$ ). We authorize only one change per aircraft, i.e. an aircraft cannot change heading and change speed concurrently. Let  $m_i^k$  be a binary variable that indicates that aircraft  $i$  executes maneuver  $k$ ,  $k \in \mathcal{M}$ . The following discrete dynamics are used for the aircraft:

$$V_i(t+1) = V_i(t) + m_i^1\Delta V - m_i^2\Delta V, \quad (1)$$

$$\theta_i(t+1) = \theta_i(t) + m_i^3\Delta\theta - m_i^4\Delta\theta, \quad (2)$$

$$FL_i(t+1) = FL_i(t) + m_i^5\Delta FL - m_i^6\Delta FL. \quad (3)$$

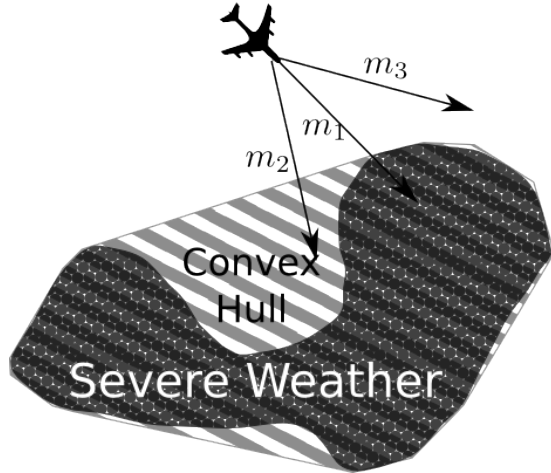
We assume the maneuvers are immediately implemented.

Theoretically, all the aircraft are able to execute all maneuvers in  $\mathcal{M}$ . Actually, all maneuvers might not be feasible for each aircraft due to weather, airspace boundary, flight level change constraints and aircraft performance restrictions. Let  $\mathcal{M}_i^{feas}$  be the set of feasible maneuvers for aircraft  $i$ .  $\mathcal{M}_i^{feas}$  is a subset of  $\mathcal{M}$  and is initially populated with all the elements of  $\mathcal{M}$ . We now present the constraints used to determine if a maneuver is feasible or not. If a maneuver is not feasible for aircraft  $i$ , it is removed from  $\mathcal{M}_i^{feas}$ .

## Heading Change

To determine if a heading change is feasible, we look at the following constraints: Weather avoidance and airspace boundary.

**Weather Avoidance** Let  $t_w$  be the time horizon for weather avoidance. A maneuver is not feasible if, after execution, the new trajectory encounters a weather cell within a time interval  $[0, t_w]$ . We model severe weather with polyhedral cells in which aircraft are not allowed to fly. Let  $\mathcal{W}$  be the set of all severe weather



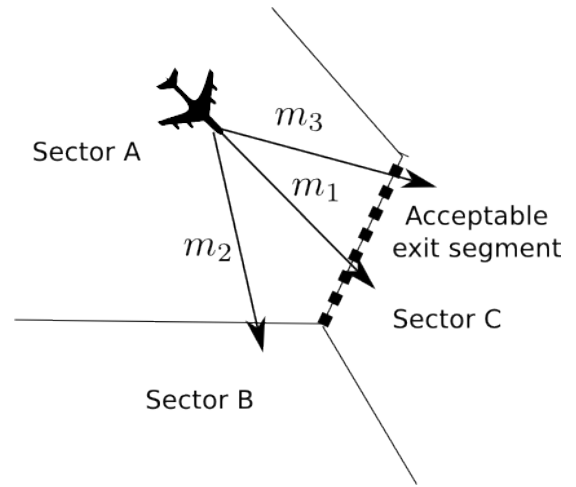
**Figure 1: Weather avoidance constraints**

polyhedra  $w_l, l = 1..n_w$ . Weather position and size estimation can be obtained or simulated [10].

To detect whether an aircraft encounters severe weather for  $t \leq t_w$ , we extrapolate the position of the aircraft for different times smaller than  $t_w$  using the above dynamics. If the extrapolated position lies within a weather polyhedron  $w_l \in \mathcal{W}$ , then the maneuver is not feasible and removed from  $\mathcal{M}_i^{feas}$ .

Weather polyhedra are 3-dimensional and do not necessarily cover all of the flight levels. They are extrapolated to their convex hull. The example depicted on Figure 1 shows the reason for dealing with the convex hull instead of the polyhedron itself. The arrows represent the resulting velocity vectors after execution of maneuvers  $m_1, m_2$  or  $m_3$ . The velocity vectors are scaled so that the arrow heads correspond to the extrapolated position at time  $t_w$ . Maneuver  $m_1$  corresponds to the nominal trajectory and is not feasible since it leads the aircraft into the severe weather cell. If  $m_2$  is executed, the aircraft would not encounter weather for  $t < t_w$  but would then have no choice but to turn back. Maneuver  $m_3$  is therefore the only feasible maneuver.

**Airspace Boundary Constraints** When proposing a heading change, one needs to be careful of sector changes. When an aircraft changes sector, the surveillance of the aircraft is handed out to another controller. If there is a surveillance degradation, this change might be difficult and should be avoided. Traffic flow management is another reason to avoid sector changes when trying to solve conflicts. If the neighboring sector is at



**Figure 2: Sector boundary constraints**

maximum capacity, a heading change leading an aircraft to this sector is not permitted. A heading change leading the aircraft to exit the sector out of the acceptable exit segment is not allowed. In the situation depicted by Figure 2,  $m_2$  is not permissible since it leads the aircraft to exit the sector out of the allowed segment, in a time less than a given duration designated by  $t_s$ .

### Speed Change

Depending on the cruise speed and altitude, accelerating or decelerating could bring the aircraft out of its flight envelope [11]. Therefore, such a maneuver will not be allowed. Flying out of the flight envelope can have dramatic consequences.

### Flight Level Change

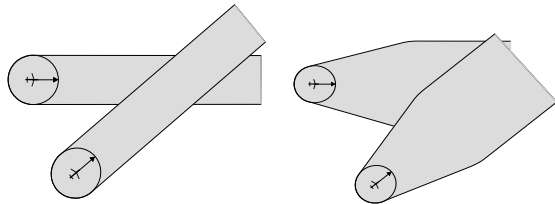
Vertical avoidance maneuvers are very efficient. They are the maneuvers used by TCAS [12]. In this paper, we aim at getting a sustainable situation after the resolution maneuver. This means we are not only interested in solving conflicts, but also in ensuring a conflict-free situation for a given period of time. Therefore, vertical avoidance maneuvers are extended to flight level changes. A flight level change is not allowed if:

- The aircraft is at its service ceiling, that is its weight does not allow it to climb higher [11].
- Changing flight level might immediately affect traffic on the crossed flight level and on the targeted flight level.
- The aircraft is about to initiate its descent.

## Degradation Modeling and Conflict Detection

So far, the set of all possible maneuvers  $\mathcal{M}_i^{feas}$  for all  $i \in \mathcal{S}$  has been determined. We shall now determine if a combination of maneuvers lead to a conflict. We first briefly introduce the model for degradation of performances and uncertainties.

Degradation is modeled as a growth of the radius of avoidance. The motivation behind this model is presented in [13]. The initial radius of avoidance is denoted by  $r_0$  and the final one by  $r_f$ . We assume the rate of growth of the radius  $\dot{r}$  to be constant, i.e.  $t_g = \frac{r_f - r_0}{\dot{r}}$ , where  $t_g$  is the time of growth of the radius of avoidance. It represents the time we allow to increase the separation distance between aircraft. We assume that the radius of avoidance has a constant value for  $t > t_g$ . Figure 3(a) presents the track of a constant radius of avoidance and Figure 3(b) presents the problem of conflict avoidance with growing radius of avoidance. We assume that the degradation does not affect aircraft vertical performances, that is traffic does not need to be spread out vertically.



a) Nominal conflict avoidance problem      b) Conflict avoidance problem with growing uncertainties

**Figure 3: Conflict avoidance problems comparison**

Let  $t_c$  be the time horizon for conflict detection. Allowing only maneuvers that do not lead to a conflict in the next  $t_c$  minutes ensures that the airspace will remain conflict free for the next  $t_c$  minutes.

To determine if a maneuver leads to a conflict, we look at the existence of a point of intersection of the circles of avoidance. If it exists, we calculate the time to conflict.

Let  $X_r$  and  $V_r$  define the relative position and velocity of aircraft  $j$  with respect to aircraft  $i$ :

$$X_r = \begin{bmatrix} x_j - x_i \\ y_j - y_i \end{bmatrix}, \quad V_r = \begin{bmatrix} v_{x_j} - v_{x_i} \\ v_{y_j} - v_{y_i} \end{bmatrix} \quad (4)$$

We first present the condition for conflict existence

with final radius of avoidance  $r_f$ . The two circles of avoidance will intersect when the distance between the two aircraft is equal to  $2r_f$ :

$$\|X_r + V_r t\| = 2r_f \quad (5)$$

The discriminant of this polynomial in  $t$  is

$$\Delta = (2X_r \cdot V_r)^2 - 4V_r^2(X_r^2 - 4r_f^2) \quad (6)$$

If  $\Delta \leq 0$ , the circles will never intersect or be tangent and there is no conflict. The corresponding maneuvers are then a feasible combination. If  $\Delta > 0$ , we have two solutions corresponding to the very first time the circles intersect and to the end of the intersection period.

$$t_{1,2} = \frac{-2X_r \cdot V_r \pm \sqrt{(2X_r \cdot V_r)^2 - 4V_r^2(X_r^2 - 4r_f^2)}}{2V_r^2} \quad (7)$$

Thus we have the following potential cases:

- $t_1 \leq t_2 \leq 0$ : The circles intersected in the past, no conflict.
- $t_1 \leq 0 \leq t_2$ : The circles of radius  $r_f$  are already intersecting. We need to compute the time of intersection for growing the circles. If  $\|X_r\| > 2r_f$ , a sufficient condition for no conflict is  $X_r \cdot V_r < 0$ .
- $0 \leq t_1 \leq t_g$ : The circles of avoidance will intersect before the radii of avoidance reach  $r_f$ . We need to compute the time of intersection of the growing circles.
- $0 < t_g \leq t_1 < t_c$ : The circles will intersect with radius of avoidance  $r_f$ , in a time shorter than  $t_c$ . This combination of maneuvers is not permitted, since it leads to a conflict.
- $0 < t_c \leq t_1$ : The circles will intersect in a time longer than the time horizon fixed for conflict detection. This maneuver is feasible.

Note that this requires that  $\|V_r\| \neq 0$ . If  $\|V_r\| = 0$ , aircraft have the same heading and the same speed. A conflict can only occur during the period of growing radii if the distance between the aircraft is smaller than  $2r_f$ .

If  $t_1 \geq t_g$ , the conflict will happen after the radius of the circles reaches  $r_f$ . Otherwise, the same reasoning

applies, except that the value of  $t_1$  is obtained by solving the following equation:

$$\|X_r + V_r t\| = 2(r_i + \dot{r}t) \quad (8)$$

which has solutions (if any):

$$t_{1,2} = \frac{-(X_r \cdot V_r - 8r_i \dot{r}) \pm \dots}{(V_r^2 - 4\dot{r}^2)} \quad (9)$$

$$\frac{\sqrt{(X_r \cdot V_r - 4r_i \dot{r})^2 - 2(V_r^2 - 4\dot{r}^2)(X^2 - 4r_i^2)}}{(V_r^2 - 4\dot{r}^2)} \quad (10)$$

If no conflict is detected, we define  $K_{ij}^{kl}$ , a binary variable indicating that aircraft  $i$  executes maneuver  $m_i^k \in \mathcal{M}_i^{feas}$  and aircraft  $j$  executes maneuver  $m_j^l \in \mathcal{M}_j^{feas}$ . Define  $\mathcal{K}_{ij} = \{m_i^k \in \mathcal{M}_i^{feas}, m_j^l \in \mathcal{M}_j^{feas}, i \text{ and } j \text{ not in conflict}\}$ . If  $K_{ij}^{kl} = 1, \forall k \in \mathcal{M}_i^{feas}, l \in \mathcal{M}_j^{feas}$ , then aircraft  $i$  and  $j$  are not in conflict, no matter what combination of maneuvers is chosen. We say that aircraft  $i$  and  $j$  do not interact. We now present a clustering method based on possible interactions to reduce the computing load.

### Clustering

To optimize the computational performance, aircraft are split into clusters  $C$ . Clustering allows parallel computation. Aircraft are clustered by potential interaction, i.e aircraft  $i$  and  $j$  belong to the same cluster if one of the combination of feasible maneuvers of  $\mathcal{K}_{ij}$  is executed and leads to a conflict. Algorithm 1 is used to cluster aircraft based on their possible interaction.

## MIP Formulation

The previous steps provide us with a set of aircraft clusters that can potentially interact, and a set of feasible maneuvers for each aircraft. To select the pseudo-optimal set of maneuvers that spreads out the traffic and avoids conflicts, we write a mixed integer program and use CPLEX [14] to solve it.

### Maneuver Unicity for Each Aircraft

Only one maneuver per aircraft is permitted, resulting in the following constraint:

$$\sum_{m_i^k \in \mathcal{M}_i^{feas}} m_i^k = 1, \quad (11)$$

for each aircraft  $i \in \mathcal{S}$ .

**Input:** Set of  $n$  aircraft and no cluster

**Output:** Set of clusters

```

foreach aircraft  $i = 2$  to  $n$  do
  foreach aircraft  $j = 1$  to  $i - 1$  do
    if aircraft  $i$  and  $j$  can be in conflict then
      if aircraft  $j$  belongs to a cluster  $C_k$ 
        then
          if aircraft  $i$  belongs to a cluster
             $C_l$  then
              | join clusters  $C_k$  and  $C_l$ ;
            else
              | add aircraft  $i$  to cluster  $C_k$ 
            end
          else
            if aircraft  $i$  belongs to a cluster
               $C_l$  then
                | add aircraft  $j$  to cluster  $C_l$ 
              else
                | create new cluster with
                  aircraft  $i$  and  $j$ 
              end
            end
          end
        end
      end
    end
  end

```

**Algorithm 1:** Clustering algorithm based on possible interaction

### Avoidance Constraint

For all combination of aircraft  $(i, j) \in \mathcal{C}_p$ ,  $K_{ij}^{kl} = 1$  implies that aircraft  $i$  executes maneuver  $m_i^k \in \mathcal{M}_i^{feas}$  and aircraft  $j$  executes maneuver  $m_j^l \in \mathcal{M}_j^{feas}$ :

$$K_{ij}^{kl} \implies m_i^k + m_j^l = 2. \quad (12)$$

Combining this implication with a unicity constraint on the choice of possible maneuvers for each combination of aircraft:

$$\sum_{K_{ij}^{kl} \in \mathcal{K}_{ij}} K_{ij}^{kl} = 1, \quad (13)$$

ensures that the requirement of only one conflict-free maneuver for each aircraft is met.

### Maneuver Cost

The cost of each maneuver is determined by the speed of its execution and the imminence of the conflict, since there is risk of collision when the maneuver is

not implemented on time. For instance, it takes less time to climb several hundred feet than accelerate 20 kts. The most efficient maneuver for conflict avoidance is a vertical change. A heading change is the second fastest maneuver to implement and a speed change is the slowest. Therefore, the cost  $c_{ij}^{m_i^k}$  for aircraft  $i$  of maneuver  $m_i^k$  to resolve the conflict with aircraft  $j$  will depend on the imminence of a conflict if no maneuver is executed, and if so, on the time to conflict  $t_1$ , in minutes.

$$\begin{aligned} \text{if no conflict :} & \quad c^{\Delta\theta} < c^{\Delta FL} < c^{\Delta V}, \\ \text{if } t_1 < 2 : & \quad c^{\Delta FL} < c^{\Delta\theta} < c^{\Delta V}, \\ \text{if } 2 \leq t_1 < 8 : & \quad c^{\Delta\theta} < c^{\Delta FL} < c^{\Delta V}, \\ \text{if } 8 \leq t_1 : & \quad c^{\Delta V} < c^{\Delta\theta} < c^{\Delta FL}, \end{aligned}$$

where  $c^{\Delta FL}$  is the cost of a flight level change,  $c^{\Delta\theta}$  the cost of a heading change and  $c^{\Delta V}$  is the cost of a speed change. The final cost for aircraft  $i$  to execute maneuver  $l$  is given by:

$$c_i^{m_i^k} = \sum_{j \in C_p, j \neq i} c_{ij}^{m_i^k} \quad (14)$$

where  $C_p$  is the cluster of aircraft  $i$ .

### Objective Function

The objective is to minimize the cost  $J$  of the selected maneuvers. The algorithm thus minimizes the cost  $J_p$ , for each cluster, subject to the avoidance uniqueness constraints:

$$\min_{C_p, \mathcal{M}_i^{feas}} J_p = \min \sum_{i \in C_p} \sum_{m_i^k \in \mathcal{M}_i^{feas}} c_i^{m_i^k} m_i^k. \quad (15)$$

subject to the avoidance and uniqueness constraints.

### Existence of a Solution

Given an initial limited set of potential maneuvers  $\mathcal{M}_i$  for each aircraft  $i$ , there might not exist a solution all the aircraft. This can be the case if an aircraft that in addition to being in conflict with another or several other aircraft, is highly constrained by weather, sector boundaries and performance. To resolve this issue, the set of potential maneuvers for aircraft  $i$  and for the aircraft in conflict, are extended, i.e a wider range of heading changes is added. Before choosing the best set of maneuvers, we check for each aircraft that there

exist several feasible maneuvers. If not, we add new maneuvers to  $\mathcal{M}_i$  and restart the algorithm. Even if there exist several feasible maneuvers for each aircraft, there might not exist a conflict free solution, which is immediately detected by the MILP solver. The same solution applies for this case, i.e increasing the set of potential maneuvers. This process is iterated until a solution exists.

## Simulation Results

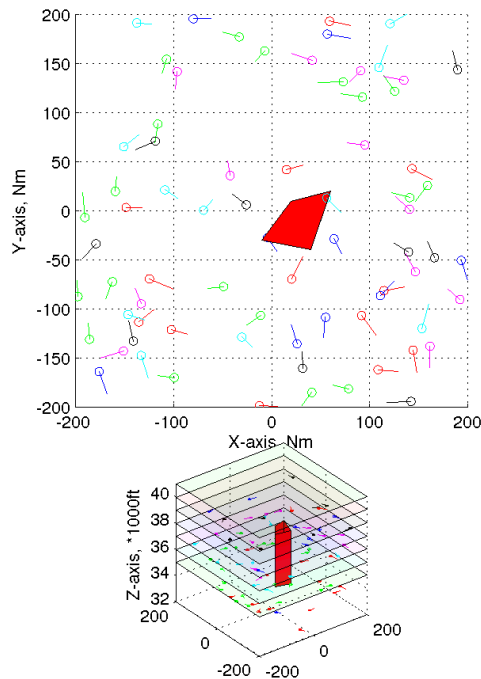
A 3-dimensional airspace, with 7 flight levels (350 to 410) was simulated. The airspace is a 400 NM sided square. Aircraft are flying on distinct flight levels but can change levels to avoid conflicts. Aircraft are flying Westbound on even-numbered flight levels and Eastbound on odd numbered flight levels. The values of the simulation parameters are presented in Table 1.

**Table 1: Parameters used for simulation**

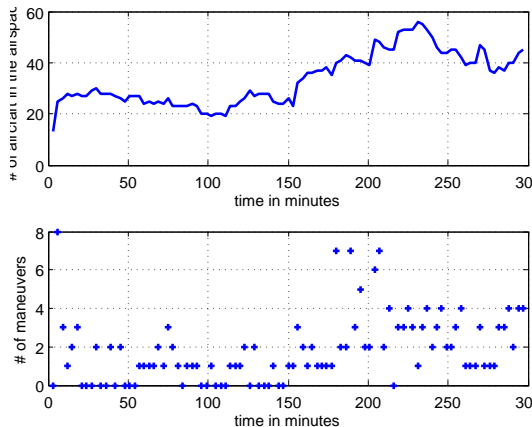
$r_0$	2.5 NM.
$r_f$	5 NM.
$\dot{r}$	50 kts.
$t_c$	For same flight level: 15 min.
$t_c$	When changing flight level: 5 min for flight level crossed.
$t_c$	When changing flight level: 15 min for destination flight level.
$t_g$	3 min.
$t_w$	20 min.
$t_s$	10 min.
$\Delta\theta$	$\pm 20^\circ$ and $\pm 70^\circ$ .
$\Delta V$	-20 kts.
$\Delta FL$	$\pm 2$ flight levels.

Figure 4 presents a screenshot of the airspace during the simulation. The polyhedron in the middle represents a weather phenomenon that should be avoided. The weather does not cover all the flight levels in this simulation so aircraft can fly over it. Figure 5 displays the number of aircraft in the airspace and the number of maneuvers executed at each time step. At  $t = 150$  min, the arrival rate of aircraft in the airspace was increased. Resolution covers every three minutes of flight and maneuvers are assumed to be instantaneously executed. As the number of aircraft in the airspace increases, the number and frequency of maneuvers required also increases.

MATLAB was used to generate the feasible solu-



**Figure 4: 2D and 3D view of the airspace during simulation**



**Figure 5: Number of aircraft and maneuvers.**

tions and cluster the aircraft in the simulations. The MIP was solved with CPLEX. The overall computation time for one iteration is approximately one minute, using a standard desktop computer. CPLEX was running on parallel machines to deal with all the clusters. There were up to 25 clusters. Each cluster is solved in a few seconds.

## Conclusion

In this paper, we presented an algorithm that enables the spread of air traffic in the event of a degradation

in the CNS system. Starting with a finite number of possible maneuvers for each aircraft, we determine a set of simple maneuvers that ensures a conflict-free airspace. Such a tool could be used as a backup system for future operations concepts. This centralized algorithm can handle hundreds of aircraft using clustering and parallel processing.

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