

# **DYNAMIC STRAIN MAPPING AND REAL-TIME DAMAGE STATE ESTIMATION UNDER BIAXIAL RANDOM FATIGUE LOADING**

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# Overview

- **Motivation and Objective**
- **Damage State Estimation**
- **System Identification Approach**
- **Experimental Setup**
- **Results**
- **Summary and Future Work**

# Motivation & Objective

**Motivation:** Automatic and **real-time** structural health monitoring and condition based life prognosis may **reduce life cycle cost** and help to **avoid catastrophic failure** of aerospace, mechanical & civil engineering structural systems.

## Objective:

Develop an SHM approach that can use strain gauge measurements to estimate damage condition of a structure under random loading

### Online damage state estimator

Based on system identification or machine learning

Current condition updating

### Future load

### Offline damage state predictor

Based on Bayesian probabilistic model

RUL

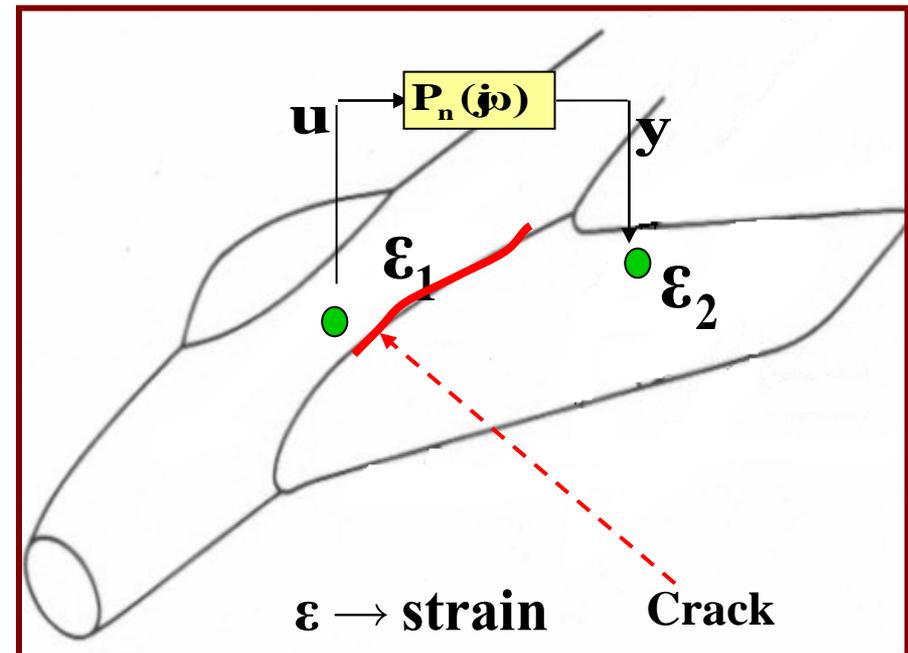
# Damage State Estimation

## Motivation for passive sensing

- Estimate local damage (Not limited to structural hot-spots)
- No external power source required
- Can use COTS sensors

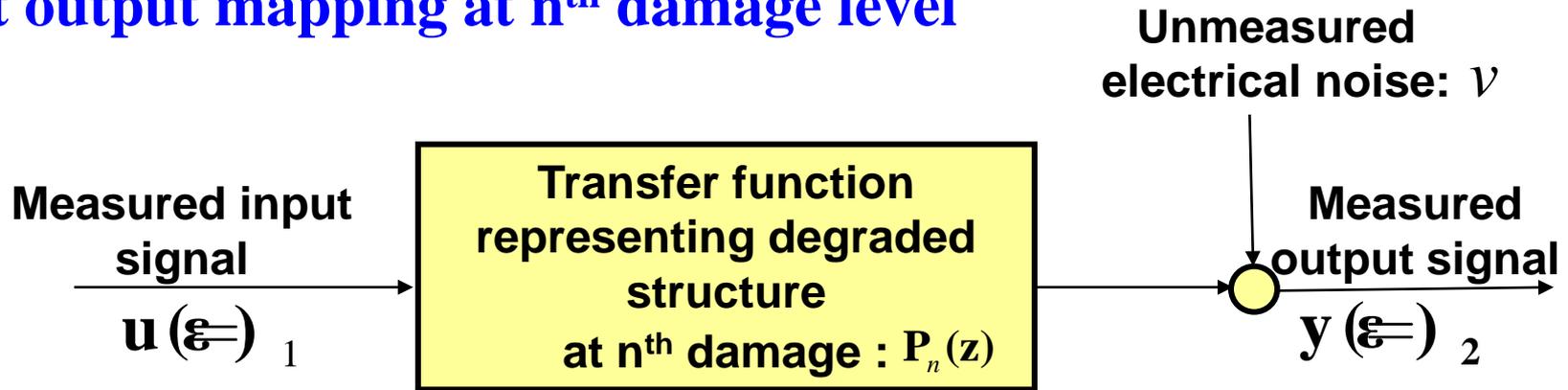
## Damage state estimation using strain measurements

- Due to damage the correlation between strain at two points changes
- Equivalent change in transfer function (TF) is a measure of change in damage states



# Motivation from System Identification

## Input output mapping at $n^{\text{th}}$ damage level



## Transfer function at $n^{\text{th}}$ damage level

$$P_n = f(R_{uy}, R_{uu}); \text{ with } u \text{ constant } P_n = f(R_{uy})$$

## Equivalent time-series damage index (for constant loading)

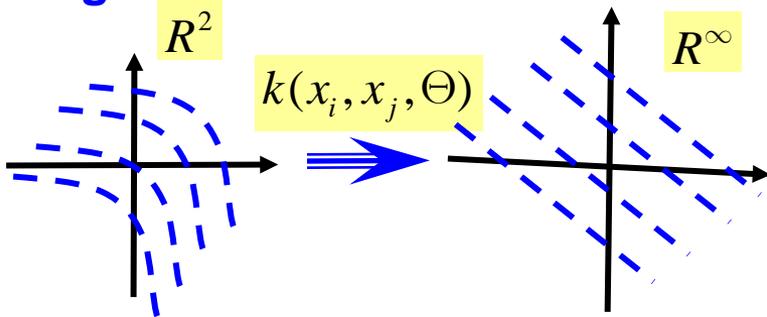
$$a_n = \sqrt{\frac{\sum_{m=0}^{m=M} ((R_{uy})_n(m) - (R_{uy})_0(m))^2}{((R_{uy})_0(m))^2}} \quad ; n = 1, 2, \dots$$

$R \rightarrow$  Correlation coefficients

# Forecasting Using Gaussian Process (GP)

- GP combination of individual distributions (assumed Gaussian)
- Input-output mapped in high dimensional space
- Conjugate gradient optimization used to estimate hyperparameters

## High dimensional transformation



- $\Theta_n^p \rightarrow$  Process
- $\Theta_n^w \rightarrow$  Input Width
- $\Theta_n^{scatter} \rightarrow$  Scatter in crack growth

## Multi layer perceptron (MLP) kernel

$$\mathbf{k}(\mathbf{x}_i, \mathbf{x}_j, \Theta_n^p, \Theta_n^w, \Theta_n^{scatter})$$

$$= \Theta_n^p \text{Sin}^{-1} \frac{\mathbf{x}_i^T \Theta_n^w \mathbf{x}_j}{\sqrt{(\mathbf{x}_i^T \Theta_n^w \mathbf{x}_i + 1)(\mathbf{x}_j^T \Theta_n^w \mathbf{x}_j + 1)}} + \Theta_n^{scatter}$$

## Negative log-likelihood function

$$L = -\frac{1}{2} \log \det \mathbf{K}_n - \frac{1}{2} \mathbf{y}_n^T \mathbf{K}_n^{-1} \mathbf{y}_n - \frac{n}{2} \log 2\pi$$

## Probability density

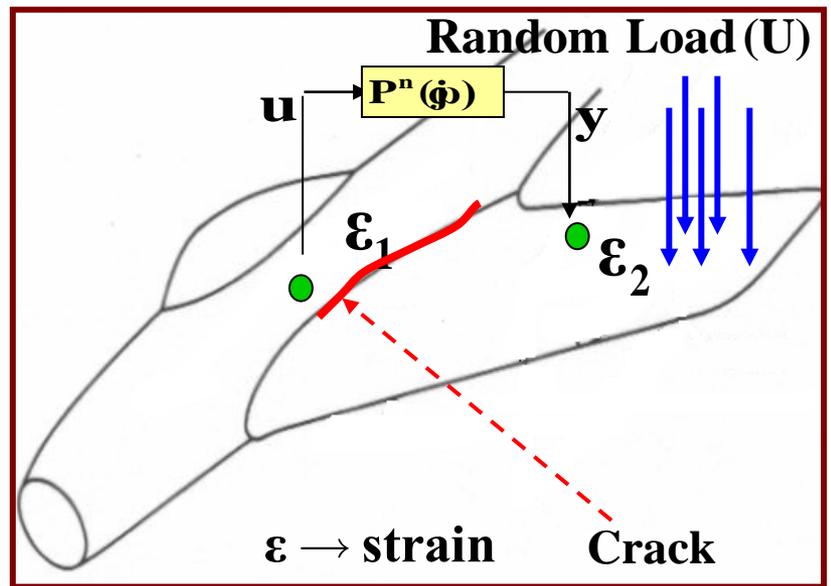
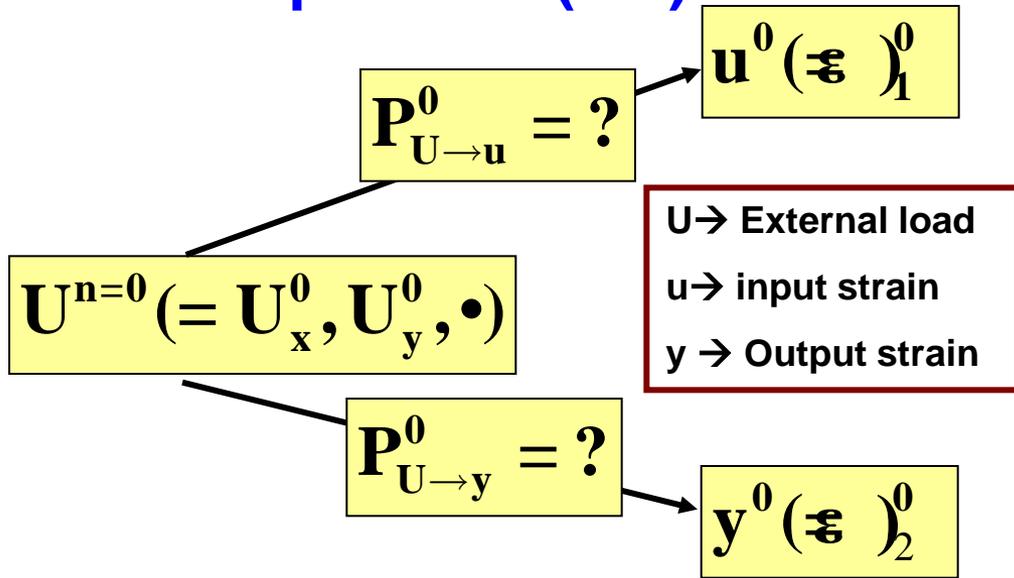
$$f(y_{n+1} | D = \{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^n, \mathbf{x}_{n+1}, \Theta)$$

$$= \mathbf{N}(\mu_{n+1}, \sigma_{n+1}^2)$$

# Dynamic Strain Based Online Damage State Estimation (Theoretical Scheme)

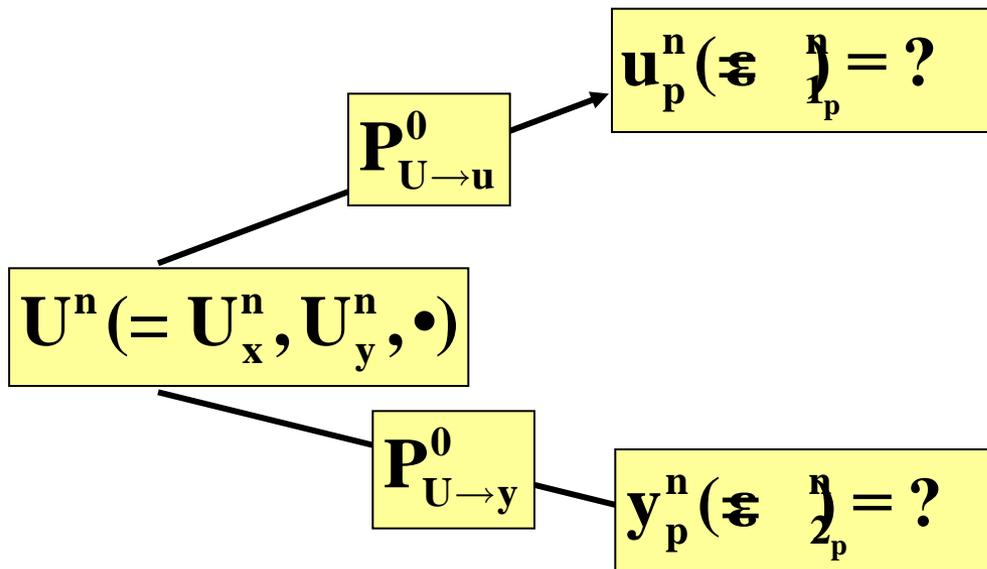
- ❑ Under random load the **change in correlation** between input (u) & output (y) can be **due to random load or due to damage**
- ❑ Need to consider loading information in damage index formulation

## Step-1: Reference Model Estimation (at n=0) using Gaussian process (GP)



# Dynamic strain mapping Based Online Damage State Estimation (Theoretical Scheme Contd.)

Step-2: Current stage dynamic strain mapping (Using GP regression)



Step-3: Current stage error signal estimation

$$e_u^n(m) = u_a^n(m) - u_p^n(m)$$

$$e_y^n(m) = y_a^n(m) - y_p^n(m)$$

Step-4: Current stage damage state

$$a^n = \sqrt{\frac{\sum_{m=0}^{m=M} (R_{e_u e_y}^n(m) - R_{e_u e_y}^0(m))^2}{(R_{e_u e_y}^0(m))^2}}$$

$R \rightarrow$  Correlation coefficient

# Experimental Setup

## Fatigue testing & data collection

**Material:** Al-2024

**Loading:** Random

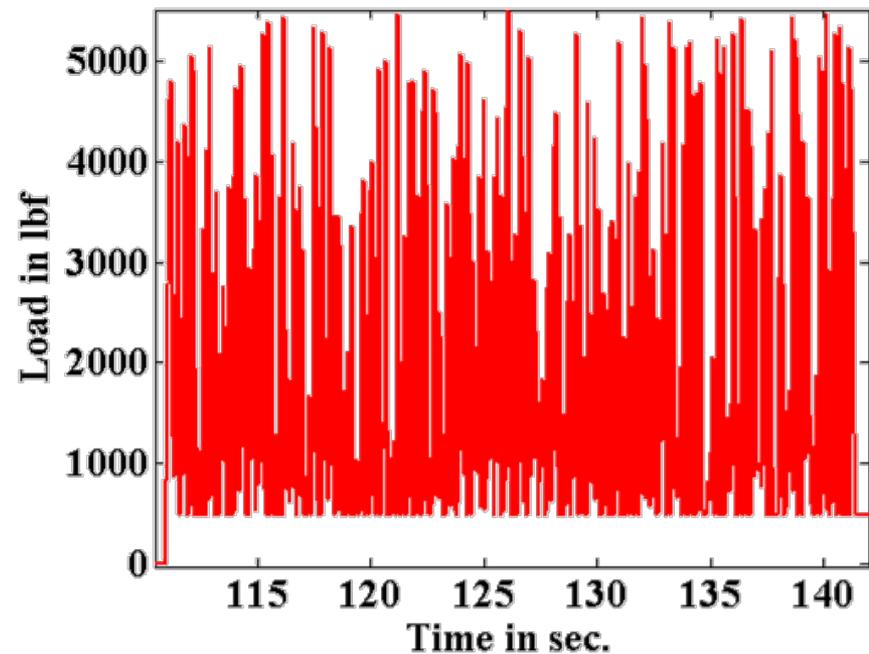
**Loading Frequency =** 10Hz

**Sampling frequency of data collection:** 1kHz

**Data collection interval:** 300 fatigue cycles

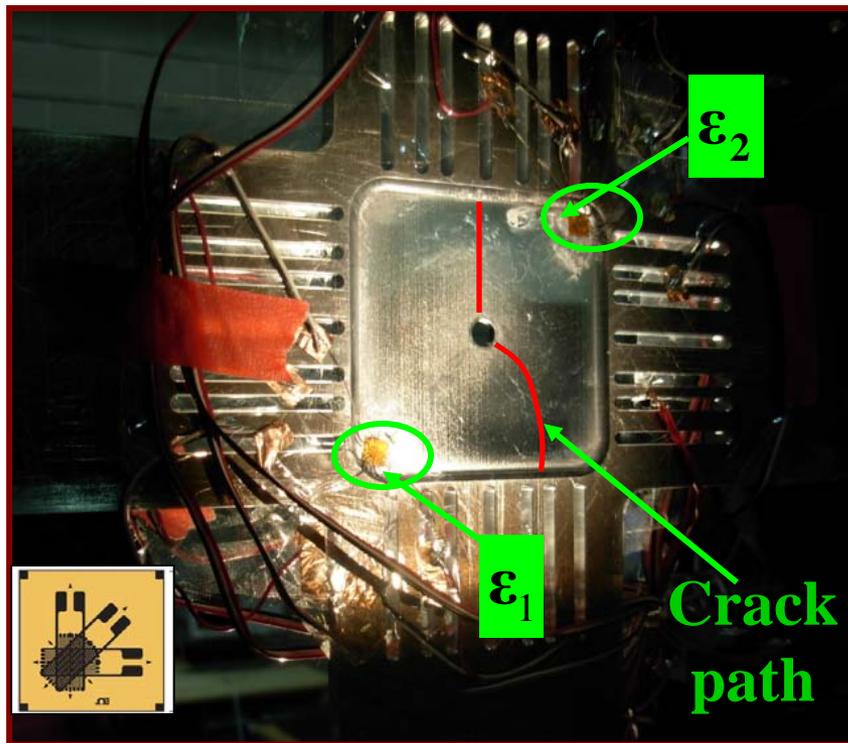


**1-block (=300 cycle) of  
random load**

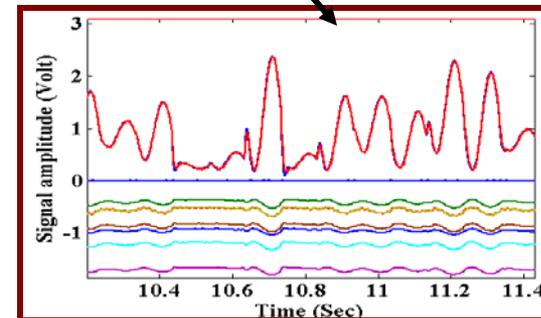
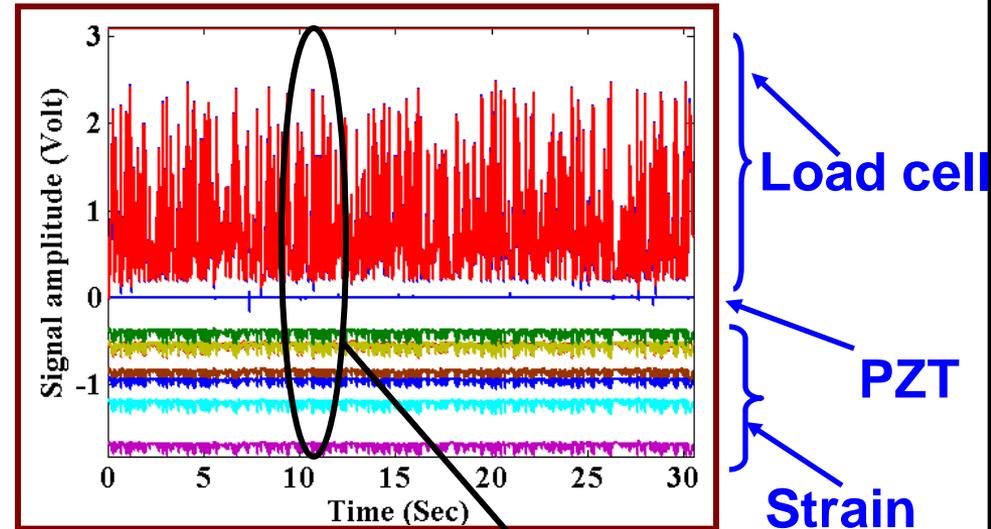


# Data Collection

## Instrumented cruciform specimen

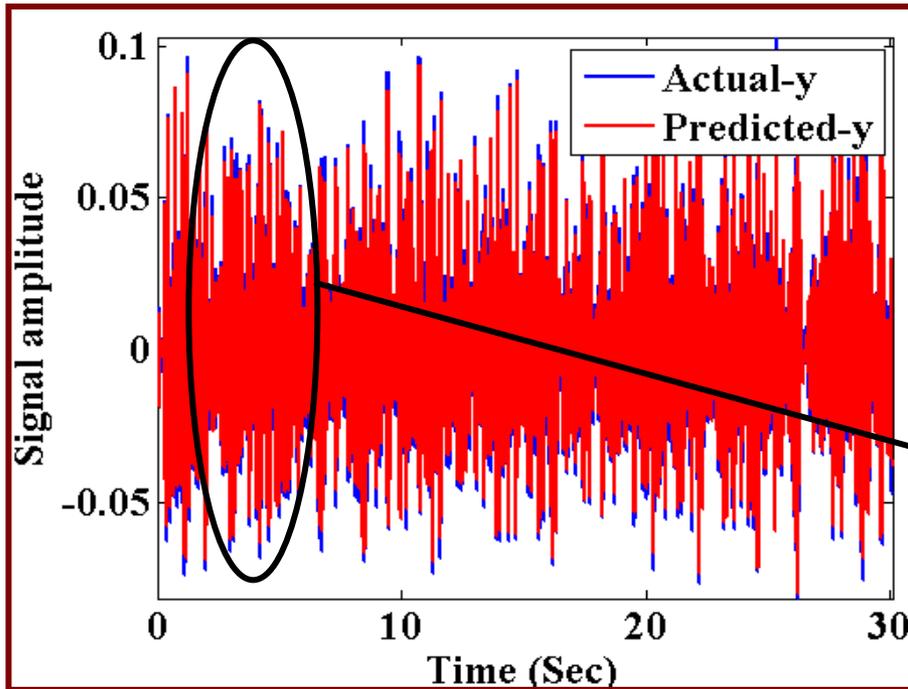


## Original signal from DAQ



# Step-1: Reference Model Estimation ( $P_{U \rightarrow u}^0$ or $P_{U \rightarrow y}^0$ ) Using Gaussian process

Comparison between regenerated (predicted) and actual strain measurement

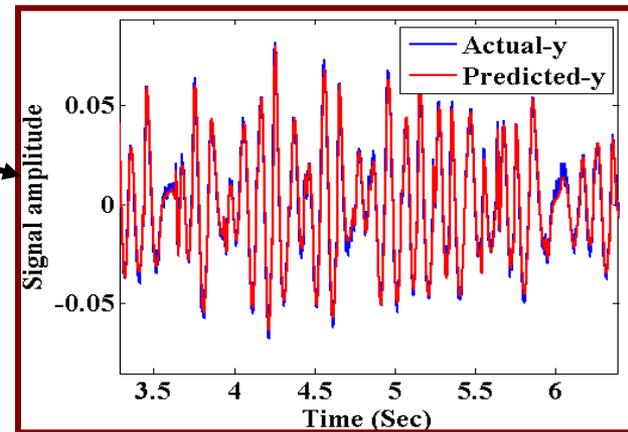


GP Input - Output

Known input =  $U_x^0, U_y^0$

Known output =  $y \xi \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

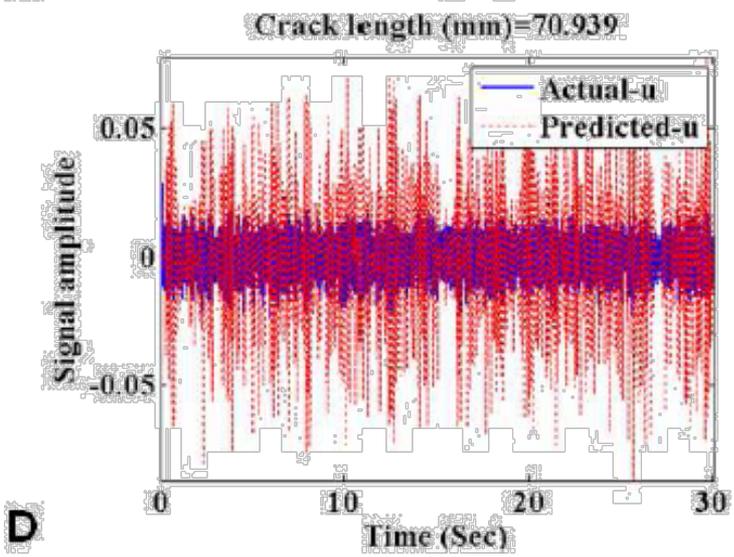
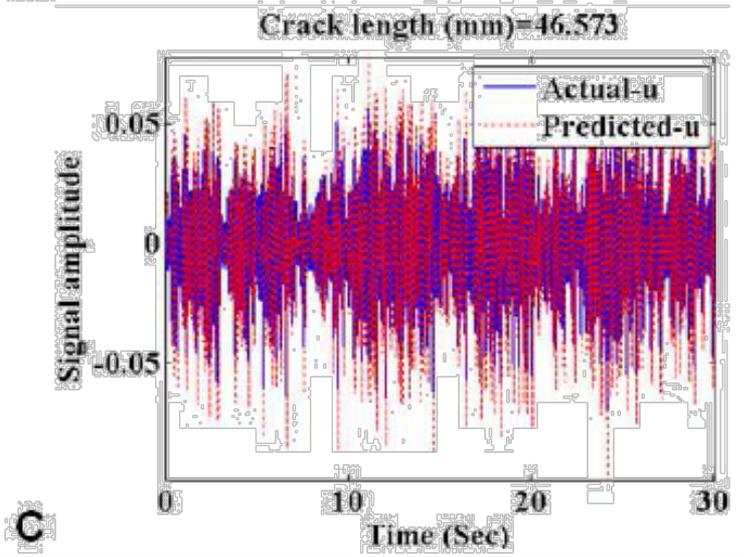
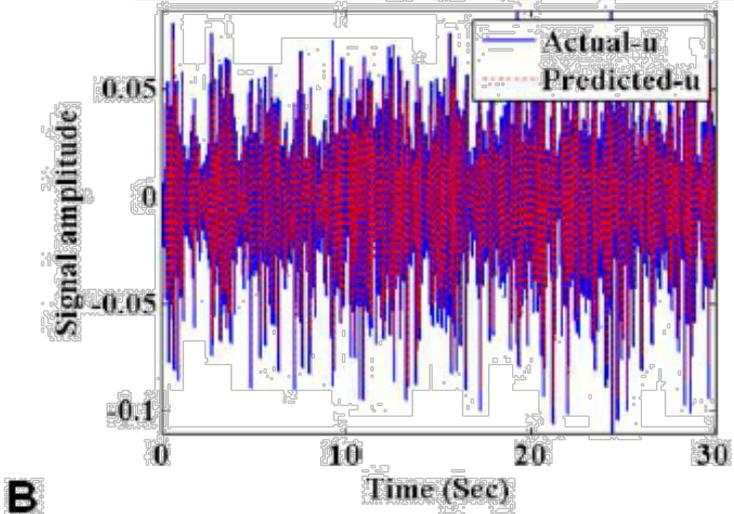
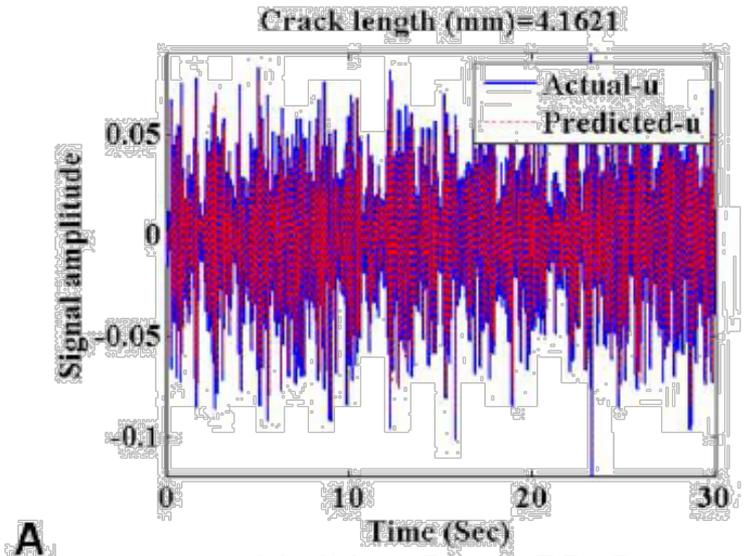
Magnified view



SHM and Prognosis

Step 2: Predicted versus actual input (u) dynamic strain at different damage levels

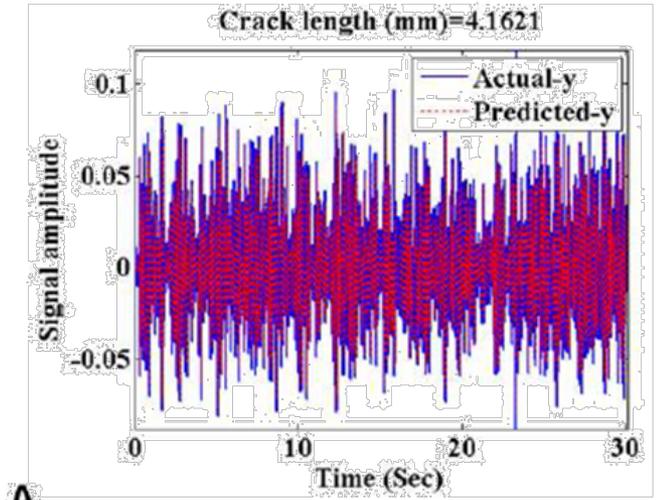
Given =  $L_x^n, L_y^n$ ; Known =  $P_{L \rightarrow u}^0, P_{L \rightarrow y}^0$   
 Unknown =  $u^n \Leftrightarrow \hat{u}^n \& y^n \Leftrightarrow \hat{y}^n$



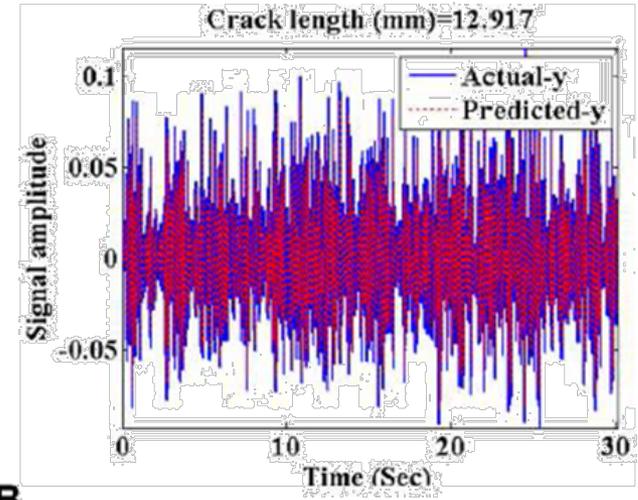
SHM and Prognosis

Step 2 (contd.): Predicted versus actual output (y) dynamic strain at different damage levels

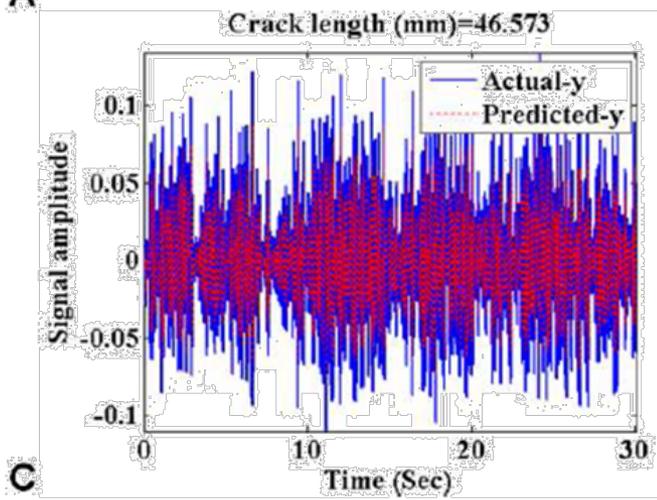
Given =  $L_x^n, L_y^n$ ; Known =  $P_{L \rightarrow u}^0, P_{L \rightarrow y}^0$   
 Unknown =  $u^n \Leftrightarrow \begin{matrix} \epsilon_1^n \\ \epsilon_2^n \end{matrix}$  &  $y = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}^n$



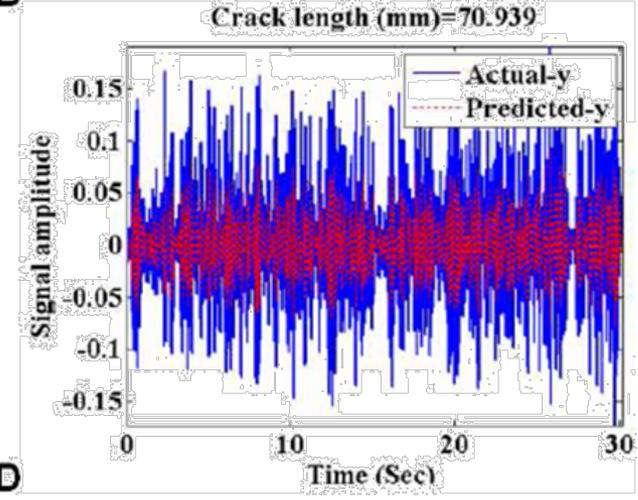
A



B

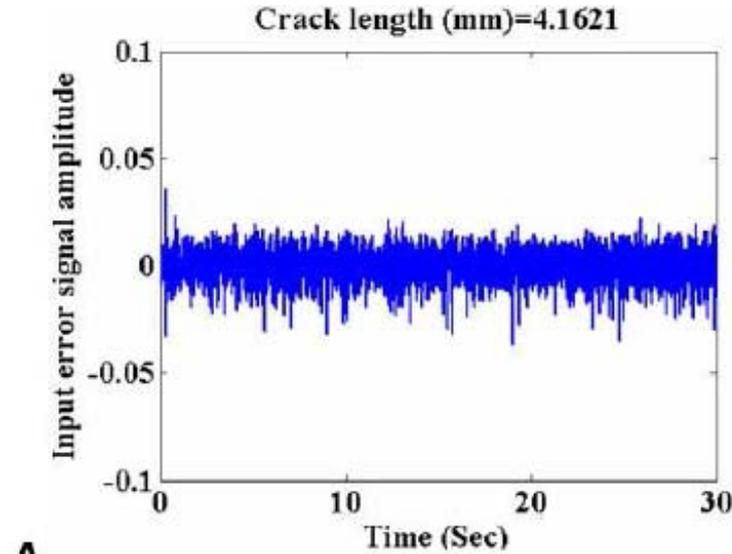


C

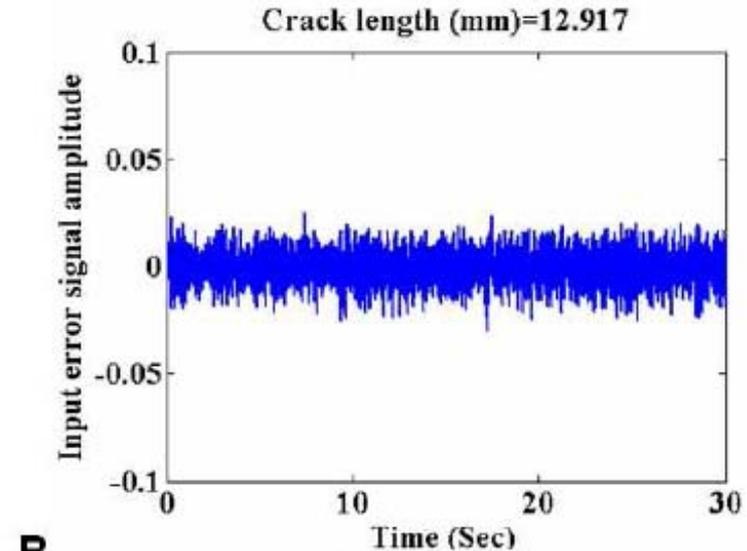


D

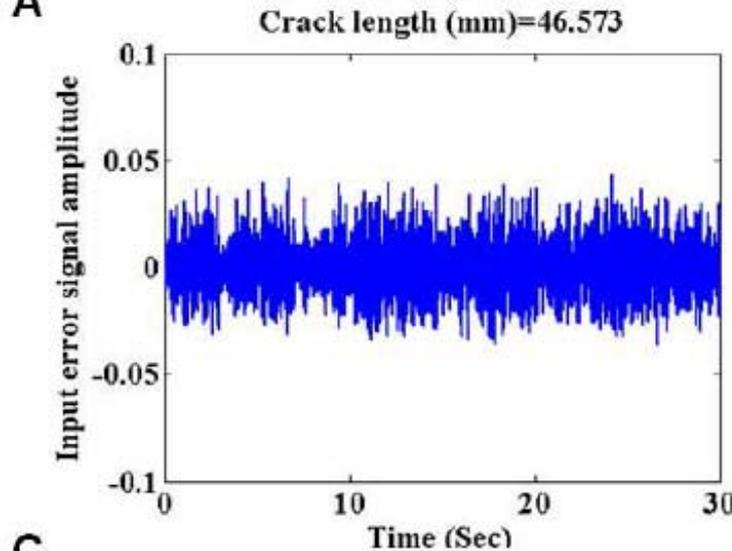
### Step 3: Time-series input (u) error signal at different damage levels



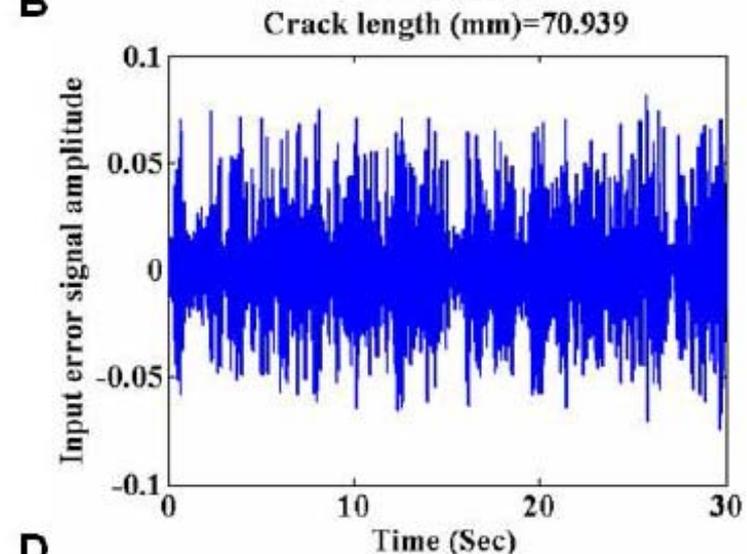
A



B

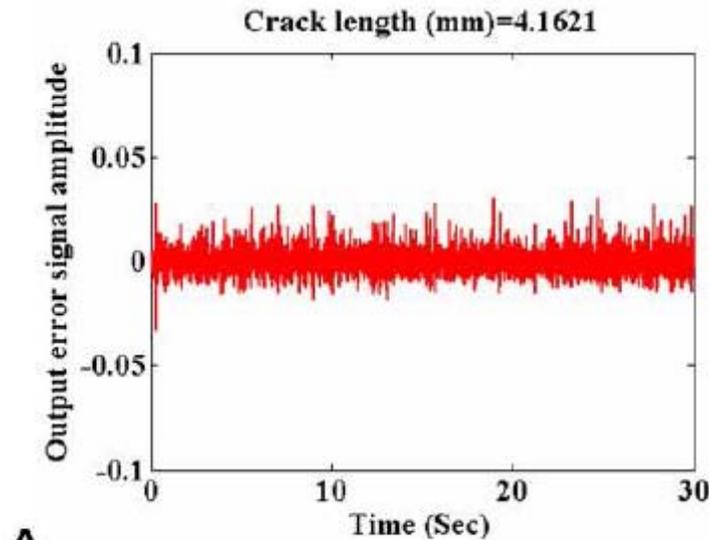


C

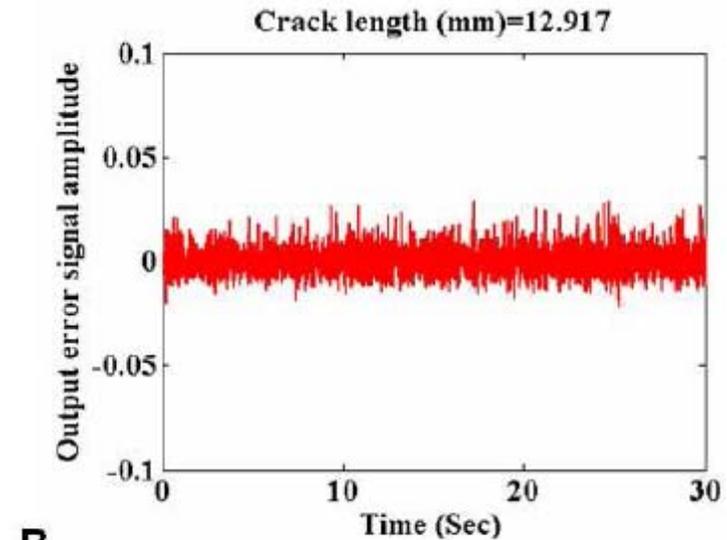


D

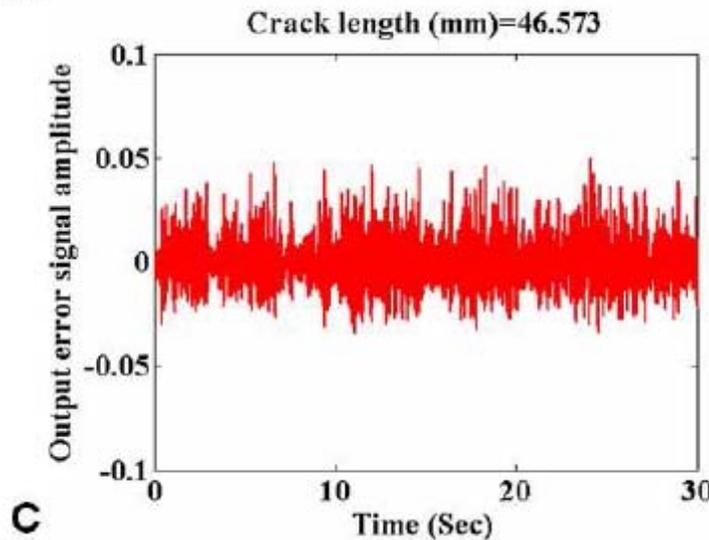
# Step 3 (contd) : Time-series output (y) error signal at different damage levels



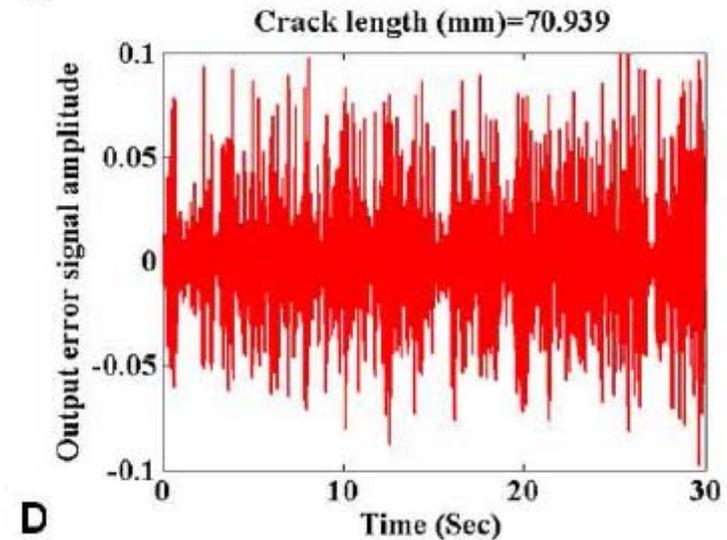
A



B



C

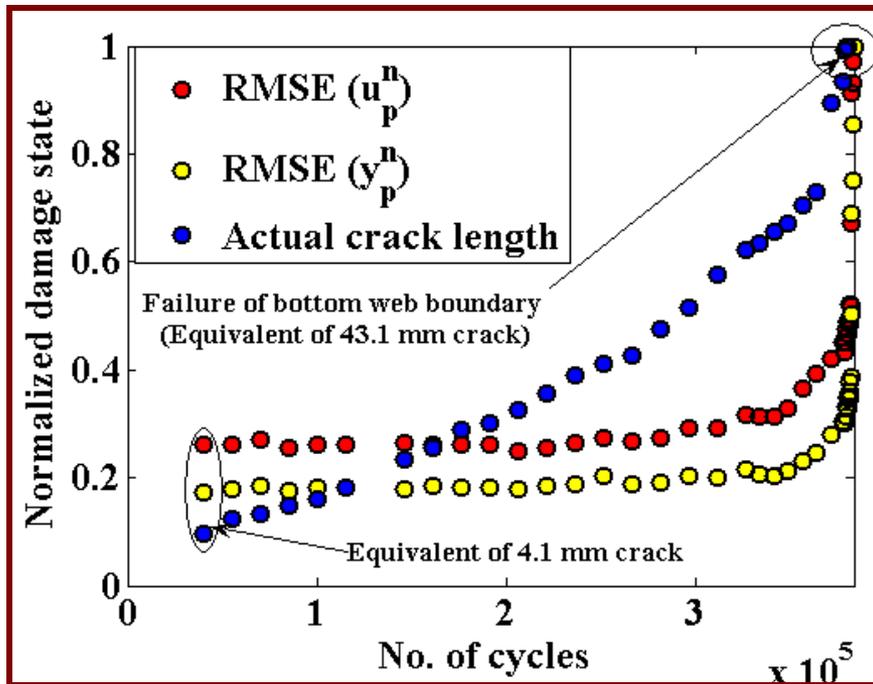


D

# Step 4 : Time-series Damage State Estimation

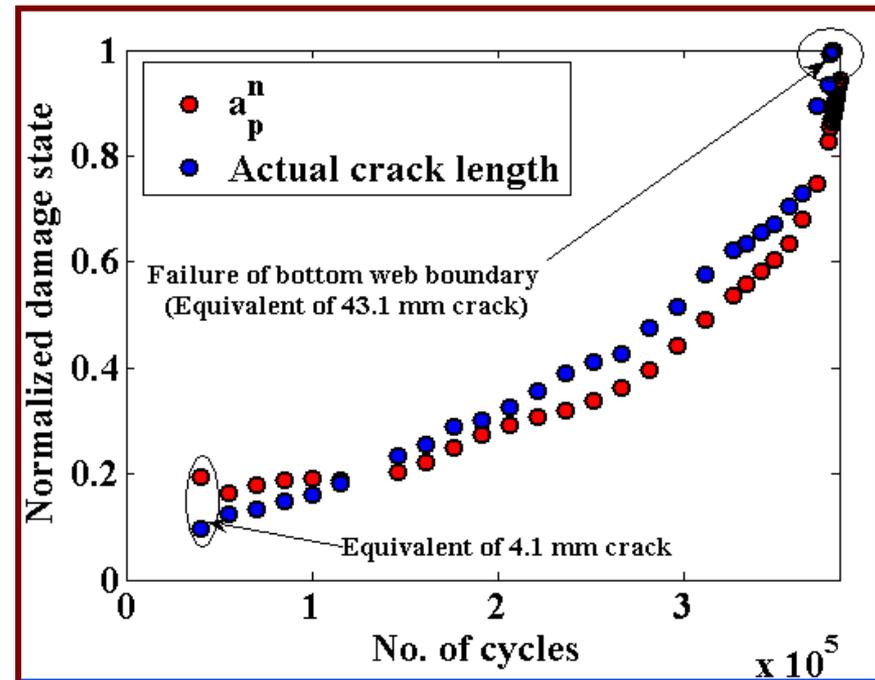
## RMSE based damage index (DI)

$$a^n = \sqrt{\frac{1}{M} \sum_{m=1}^{m=M} [e_{(u \text{ or } y)}^n(m)]^2}$$



## CRA based damage index (DI)

$$a^n = \sqrt{\frac{\sum_{m=0}^{m=M} (R_{e_u e_y}^n(m) - R_{e_u e_y}^0(m))^2}{(R_{e_u e_y}^0(m))^2}}$$



- ❑ Good correlation between visual measurements and DI time-series
- ❑ CRA is better than RMSE of predicted error signal

# Summary & Future Work

## Summary

- Applications of dynamic strain mapping model presented for **online** damage state estimation using **passive sensing**
- Gaussian process used to create input-output model
- Approach demonstrates clear trend over the entire stage II and stage III damage regime

## Future work

- More testing on different geometries
- Test using out of phase or independent random load on each axis
- Investigate alternative passive sensors to try and detect stage I cracks
- Implementing multisensor information